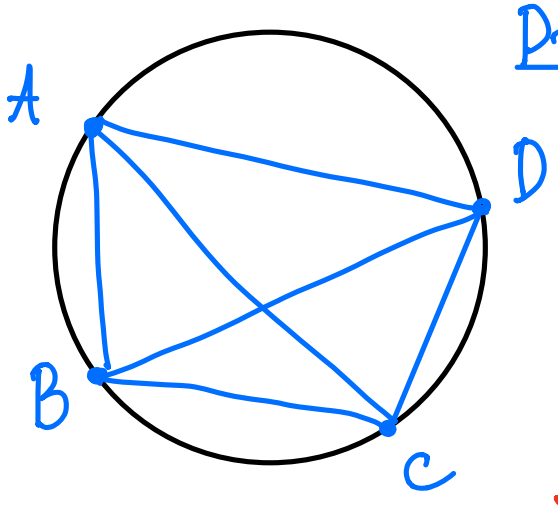
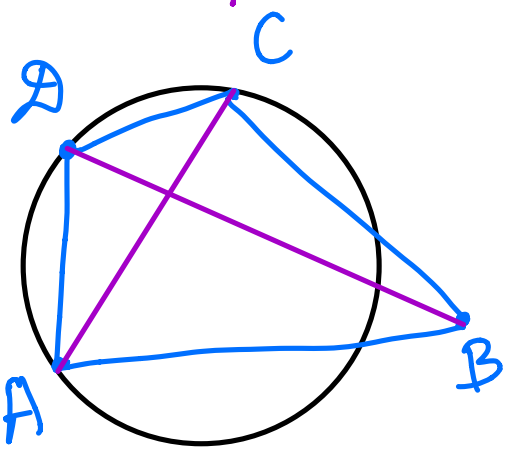


Ptolemy's Theorem

$$AB \cdot CD + BC \cdot AD = BD \cdot AC$$

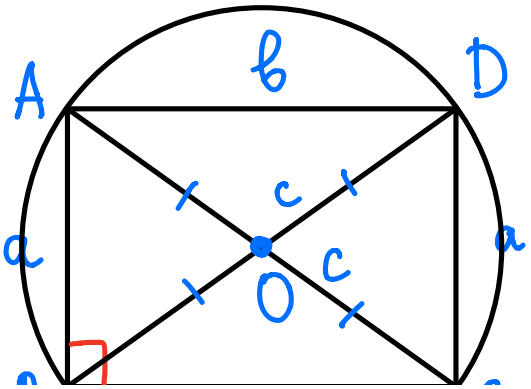
- A, B, C, D: lie on a circle
- $ABCD$: quadrilateral cyclic (vertices lie on a circle)

The sum of the products of the opp. sides of a cyclic quadrilateral = the product of the two diagonals.



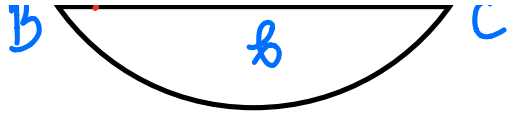
$$AB \cdot CD + AD \cdot BC > AC \cdot BD$$

Ptolemy's Inequality.



Pythagorean Theorem

$$a^2 + b^2 = c^2$$



ABCD : rectangle

By Ptolemy's Thm:

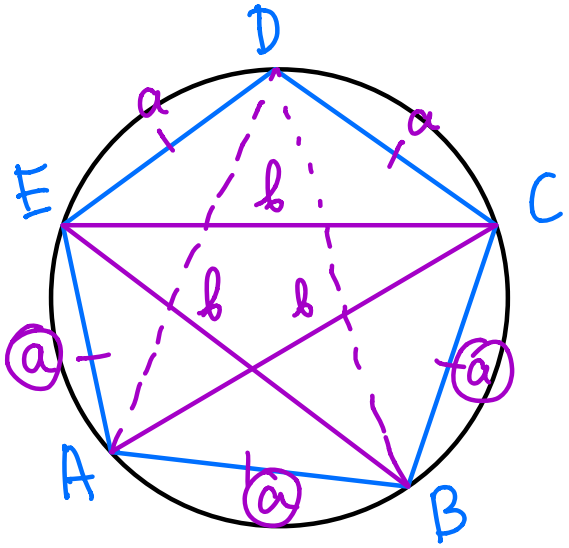
special case

$$AB \cdot CD + BC \cdot AD = BD \cdot AC$$

$$\boxed{a^2 + b^2 = c^2}$$

$\Rightarrow$ Pythagorean Thm.

Ex.



• ABCE

$$a \cdot a + a \cdot b = b \cdot b$$

$$\boxed{a^2 + ab = b^2}$$

$(\div a^2)$

$$1 + \frac{ab}{a^2} = \frac{b^2}{a^2}$$

$$\varphi = \frac{1+\sqrt{5}}{2} = \boxed{r = \frac{b}{a}}$$

$$\Rightarrow 1 + \frac{b}{a} = \left(\frac{b}{a}\right)^2$$

r $\quad$ r^2

$$\boxed{1 + r = r^2} \rightarrow \boxed{\text{quadratic eq'n}}$$

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

- x = unknown
- a, b, c : constant
- quadratic

$1+r-r^2=0$

$1\cdot r^2-r-1=0$

$a=1 \quad b=-1 \quad c=-1$

$$x=\frac{1\pm\sqrt{(-1)^2-4\cdot(1)(-1)}}{2}=\frac{1\pm\sqrt{5}}{2}$$

$\frac{1+\sqrt{5}}{2}>0$ golden ratio

$\frac{1-\sqrt{5}}{2}<0$ evil twin

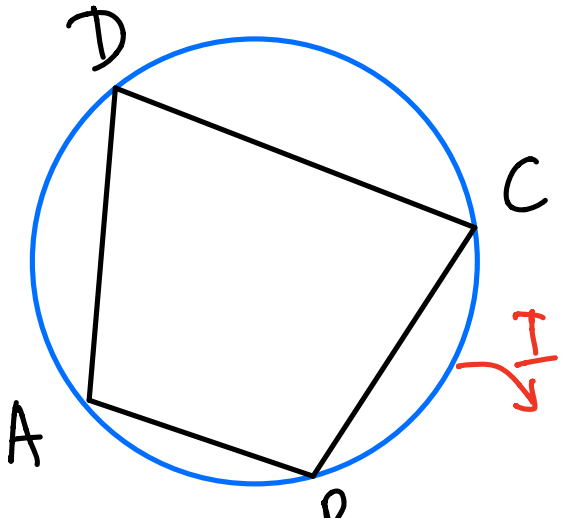
φ $\bar{\varphi}$

$ax+b=c$

linear

$x=\frac{c-b}{a}$

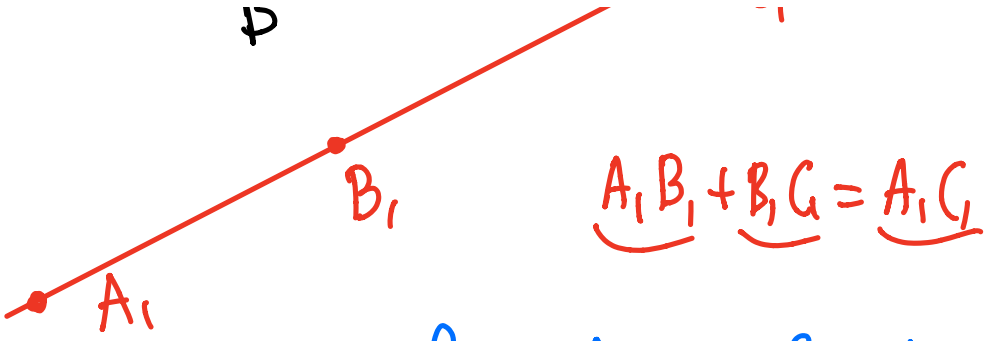
one root



Inversion in Plane

circle $\xrightarrow{I}$ line

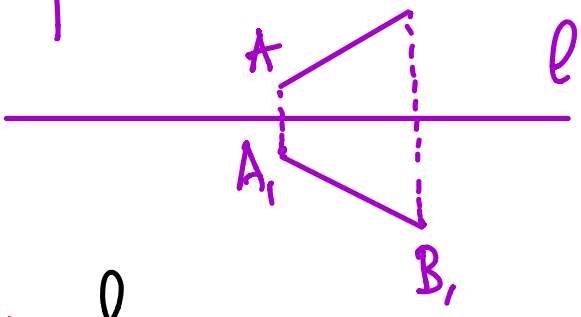
4pts $\longrightarrow$ 3pts

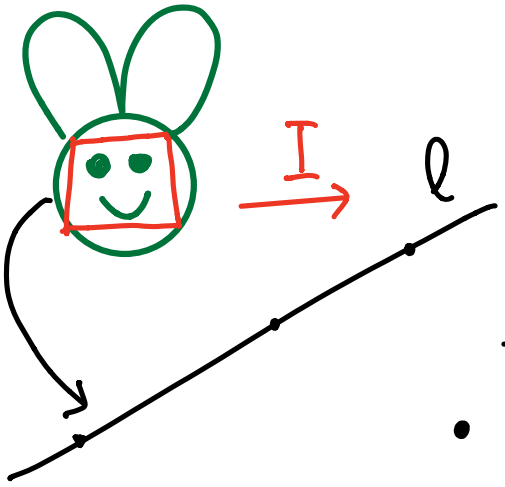


$A_1B_1 + B_1C_1 = A_1C_1$

Transformation of the Plane

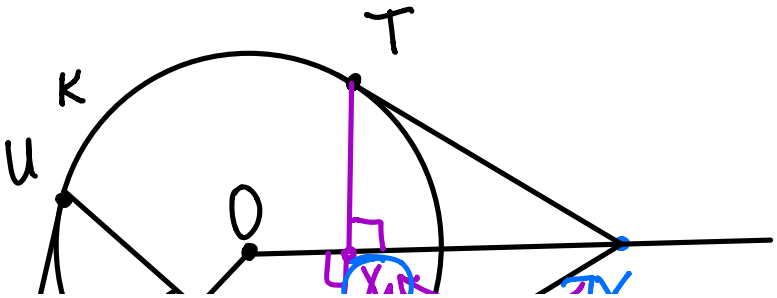
- dilation (homothety) zoom
- rotation
- translations
- reflection

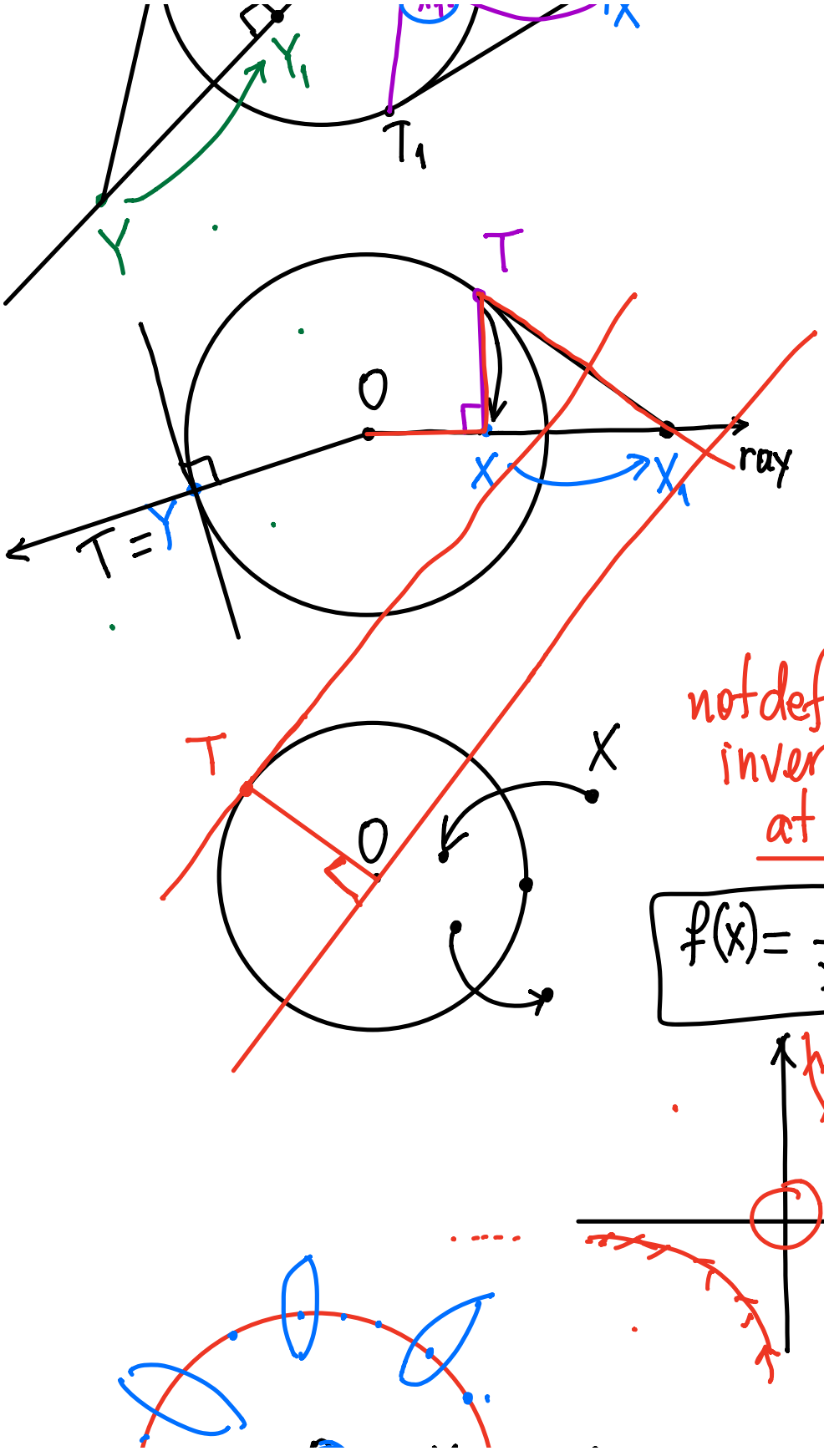




Definition of Inversion

- O, R, k : fixed

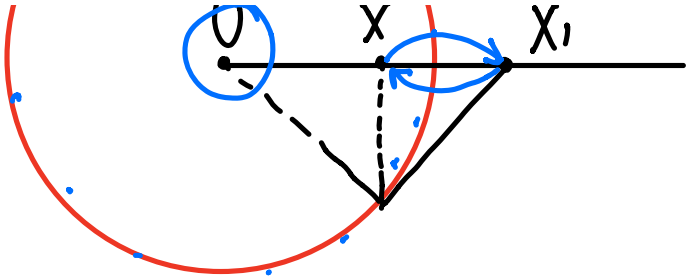




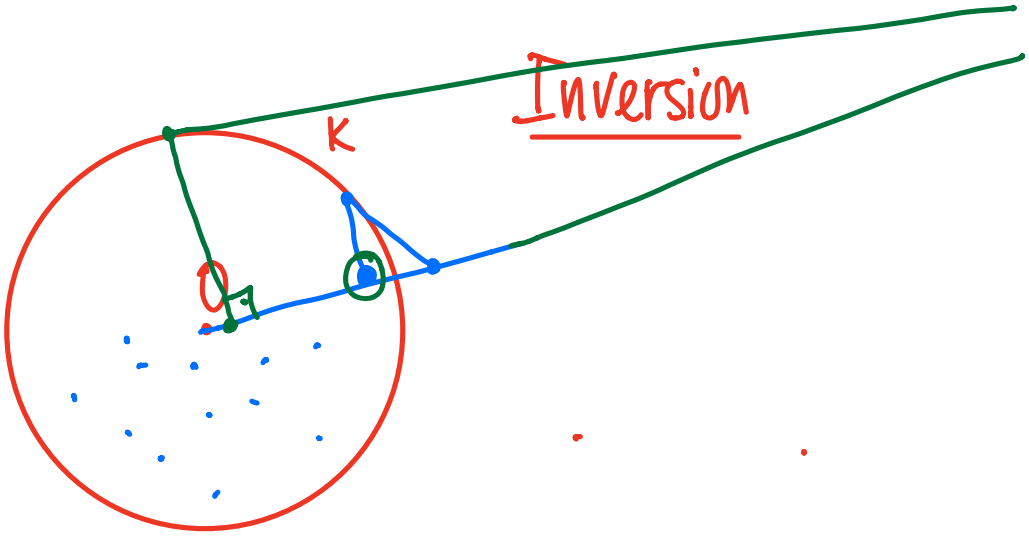
not define inversion at center O.

$$f(x) = \frac{1}{x} \qquad x \neq 0$$

4 quadrants



Q: How to catch all lions in Africa?



HW: Numberphile "Ptolemy's Thm" → main, extras