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Berkeley Math Circle: some probability riddles

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1 Additivity of expectations

1. A needle of length 1 inch is thrown randomly on the floor. On the floor are parallel lines spaced 2 inches apart. What is the probability that the needle crosses a line? If the needle has size 10 inches, what is the expected number of crossings?

   You now attach two needles of length 1 together so as to form an angle of 60 degrees. If you let it fall, what is the probability that it crosses a line? What is the probability that it crosses a line twice? What if the two needles are attached at their center and form a 90 degrees angle?

   You make a rigid square with 4 needles of length 1 inch each. If you let it fall on the floor, what is the probability that it crosses a line on two opposite sides?

2. In a far away country, factory workers get one day of holiday each day that is the birthday of any worker in the factory (and no other holidays). You want to open a factory and can hire a number of workers, without knowing their birthdays. How many workers should you hire if you want to maximize the number of man-days worked per year?

2 Random walks

3. You are standing on the middle rung of a ladder that has 21 rungs. You have a fair coin which you flip at each step. If the coin lands on head, you go up one rung, and if it lands on tails, you go down one rung. If you go up from the top rung, you exit the ladder successfully. You fail if you step down from the bottom rung. What is the probability that you successfully make it to the top? What if you start on the 5th rung from the bottom?

4. You are now on a ladder that continues infinitely below you and has 20 rungs above you. You move according to the same rule. What is the probability that you eventually make it to the top?

5. You are standing on the middle rung of a ladder that has 21 rungs. You have two coins: one is a fair coin and the other is a coin with either two heads or two tails. You know which coin is which, but you will not know if the second coins has heads or tails before you flip it. At each step, you can choose one coin and flip it. If it lands on head, you go up one rung and if it lands on tails, you go down one rung. You win if you get to the top of ladder and loose if you hit the bottom. What is your best strategy?

3 Choosing a husband

6. A princess is looking for a husband. There are fifty noble bachelors from neighboring countries wishing to marry her. Each in turn gets to spend one day with the princess. At the end of the day, the princess must make her decision, either to marry the suitor, or send him back. She cannot change her mind later. Also, she must choose one of the fifty men. Assume that after one day, the princess is able to accurately rank the suitor compared to all the ones she has seen so far. She wishes to maximize her chance of marrying the best one of all. What should her strategy be?