

BMC 2020 Advanced Darts and Balls - A Game to Always Win

I. Warm-Up/Pre-reading

1. What is the volume of a sphere (technically a 3D ball)? Can you derive the formula (probably from calculus)?
2. If you do not know calculus, here is a brief picture to explain:



Circle



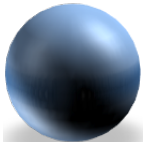
Rings



Slices



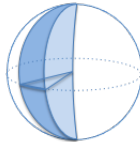
Boards



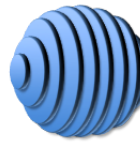
Sphere



Shells



Wedges



Plates

- a) In the picture above, the Circle has radius 4. What is its area?
- b) The picture describes a few ways to approximate the area of the circle (if we don't already know the formula) by cutting the circle into pieces whose area we know how to calculate. Suppose we cut the circle into 8 *Boards*, starting from the right, what is approximated area of the circle?
- c) The idea of calculus is to increase the amount of boards (rectangles) to get better approximation, and the limit of this approximation as we increase the number of boards to infinity will be the actual area of the circle.
- d) Repeat this process to find volume of the sphere: You should spend some time thinking about cutting a sphere into pieces (we call them discs - the picture calls them plates). What shape is each disc in? What is the volume of each disc?

II. A Dart Game

III. Balls of high-dimension