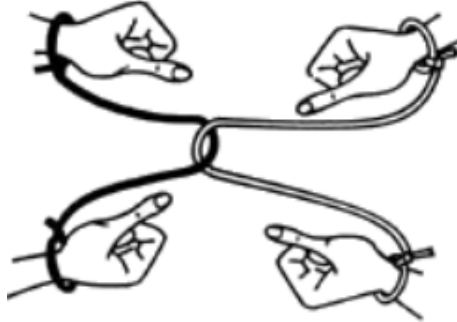# Berkeley Math Circle
April 10, 2019

1. **WARM UP:** Suppose that you and a partner have your hands handcuffed together as below. Can you untangle your hands without breaking the ropes?

2. Take a strip of paper, put a few twists in it, and tape the ends together. How many sides does the resulting surface have? Try a few different times with different number of twists.

3. Take a strip of paper, put one twist in it. This is called a möbius band.

   (a) How many sides does this surface have?

   (b) Now, cut your surface down the middle. Explain what happens. (I know you did this a few weeks ago!)

4. Now make a möbius band, and cut it a third of the way down from the top, all the way around. Explain what happens.

5. If you hang a picture on a wall with one nail, if you remove the nail then the picture falls. If you use two nails and hang the picture as below, if you remove just one nail the picture will remain hanging on the wall. Can you arrange the string holding the picture up around two nails so that if either nail is removed the picture falls (and so that it doesn't fall as long as both nails remain in the wall)?


What about with three nails?

6. Consider the two surfaces below. Imagine they are made out of play-doh or some kind of infinitely stretchy plastic so that you can you stretch, squeeze, and move them around however you like without tearing them or passing them through themselves. Can you somehow transform the left surface into the right surface?
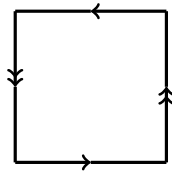
7. Consider the surface below:
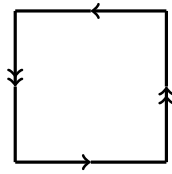
   (a) How many sides does this surface have?

   (b) What surface do you get when you cut this surface down the middle?

8. Consider the surface given by this diagram:

   Warning: We can't do this gluing in three dimensional space!

   An alien lives on this surface, and they have a rope to tie around the surface. They does this by looping it once across the middle and tying the ends together. The alien pulls on the rope, and it doesn't go anywhere.

   They then decide to make it even stronger by looping it twice across the middle and then tying the ends together. They again pulls on the rope, and without the rope breaking, they are able to pull all of the rope to where he is standing. Explain how this could have happened!