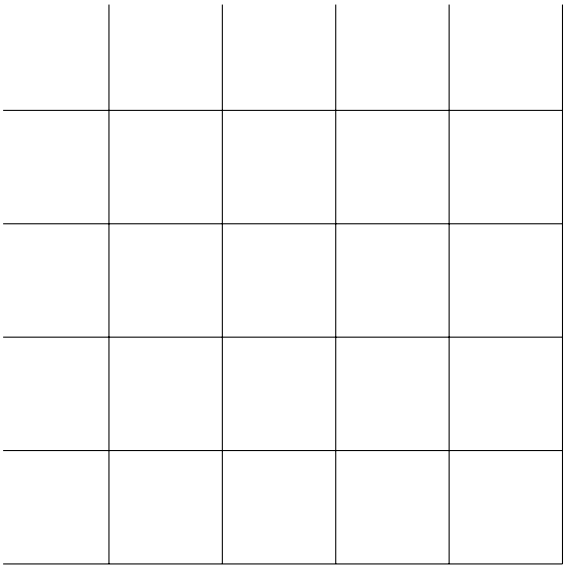
Berkeley Math Circle

April 2, 2019

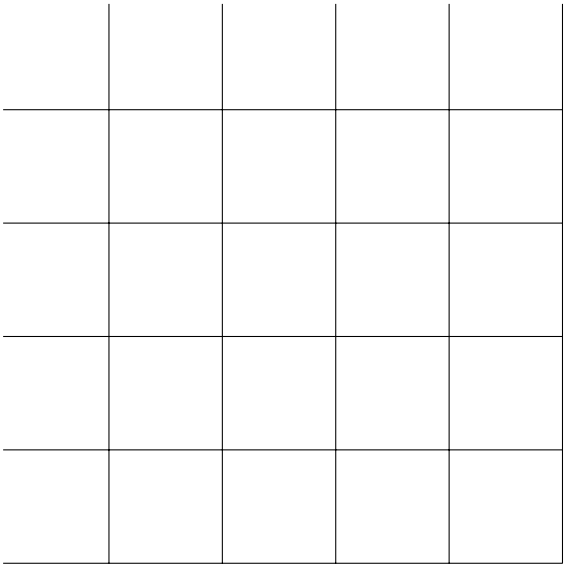
1. Take a rectangular card. How many ways can we move this rectangular card around in three dimensional space so that it fits in the same spot? We say two moves are *the same* if they have the same ending position. Give each of these moves a name!

2. Now that we understand all of the possible moves, let's try to understand what happens when we do one move and then another one. Together, let's fill in this table. We will label the columns with the different moves and fill in the grid with what happens when we do the move on the right followed by the one above it.



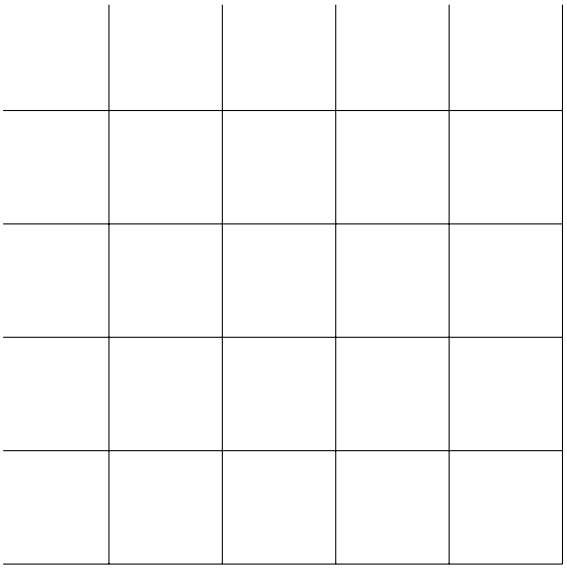
3. Now, take a square paper. How many symmetries does this square have if you are only allowed to rotate it (and **not** flip it over)? Give each of these moves a name.

4. Fill in this grid for the square the same way we did for the rectangle.



5. Is it possible to rename or rearrange the moves you can do on the square and the rectangle so the tables are the same?

6. Challenge: can you come up with any object with four symmetries that has a **different** symmetry table? *Different* means that there is no way to rename or rearrange the moves in the table to match one we have here.



7. Now, with a partner, take a pencil and three strings, and poke three holes in your rectangular card. Tie the strings to the pencil and thread them through the holes in the paper, tying a knot to hold them in place. How many symmetries does this rectangle have? You are allowed to flip and rotate the rectangular card, and as before, two moves are the same if the card and the strings look the same. You are allowed to hold the card fixed and move it around in space (without rotating or flipping) in order to simplify the strings.

   Come up with names for the moves you can do on the card, and come up with a table listing what happens when you first do one move and then another.

8. What do you notice about this table? Make some observations!

9. Now take the square paper, and look at symmetries where you can flip the square as well as rotate it. Is this set of symmetries that same one coming from cards with strings attached? Do the same thing with an octagon, and look at symmetries where you can only rotate the octagon. Is this the same as any previously encountered symmetries?