



# Board Tiling, Chocolate Breaking with a Hint of Fibonacci

Part I

By Harry Main-Luu

# General Overview

Part 1: Tiling a Plane

Part 2: Tiling a Board

Part 3: Breaking and Sharing Chocolate

Some overarching questions for today's sessions:

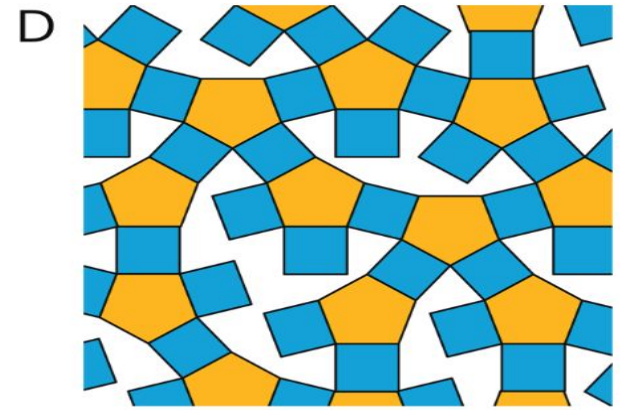
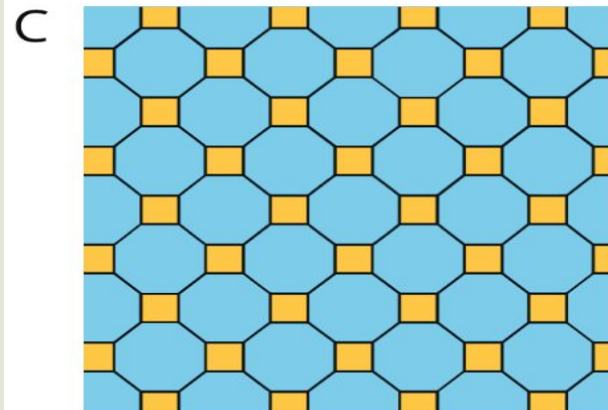
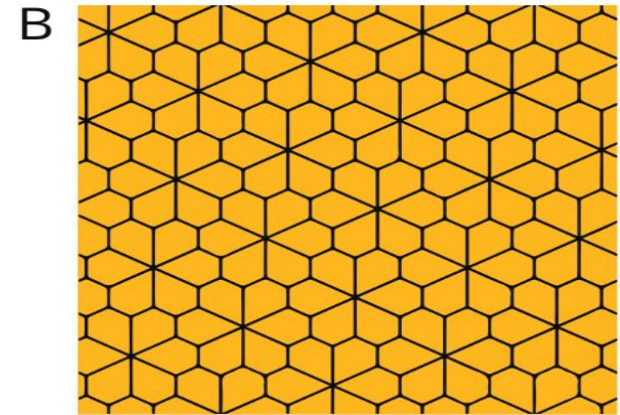
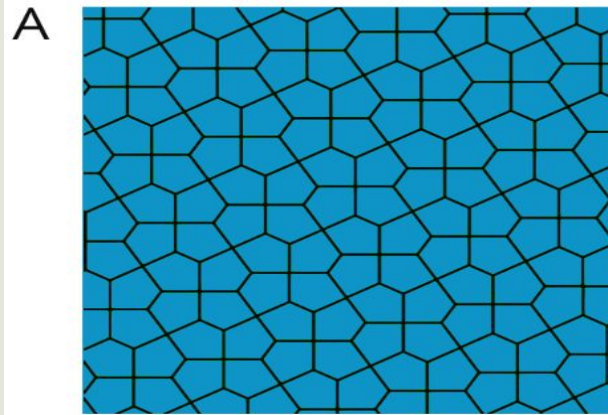
1. Which types of patterns work to tile a plane (or a floor in your bedroom?)
2. Which special shapes can tile a plane and WHY?

Some materials are adapted from U. Waterloo, Canada



# Tiling a Plane: Warm-Ups

Choose the odd one out!

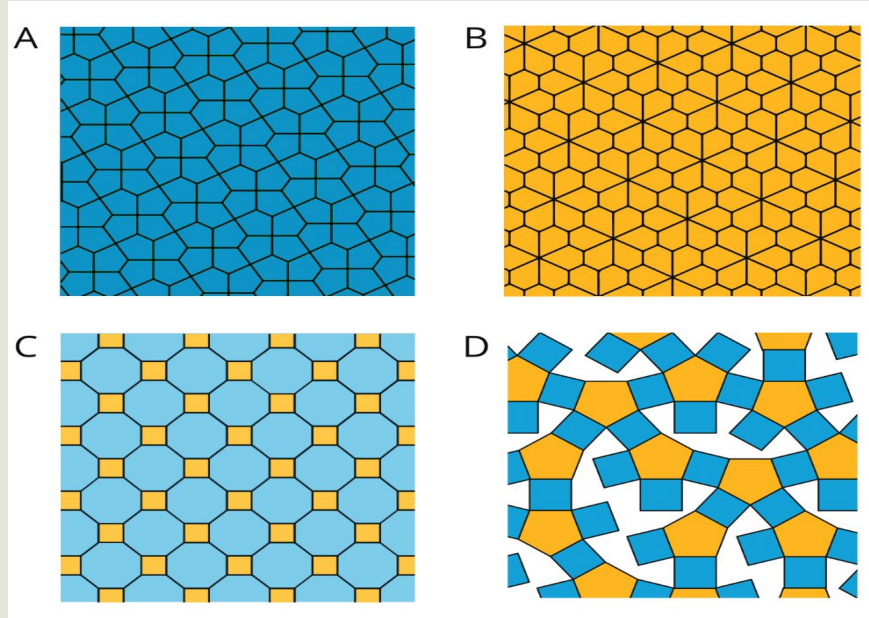


# Tiling a Plane: What is it?

**Definition:** A *tessellation* of a the plane is a way to cover the entire plane using **finitely many** types of geometric figures, where there are no overlapping or gaps.

Examples: A, B, C (earlier).

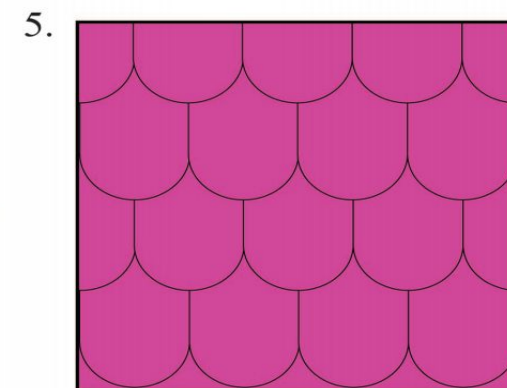
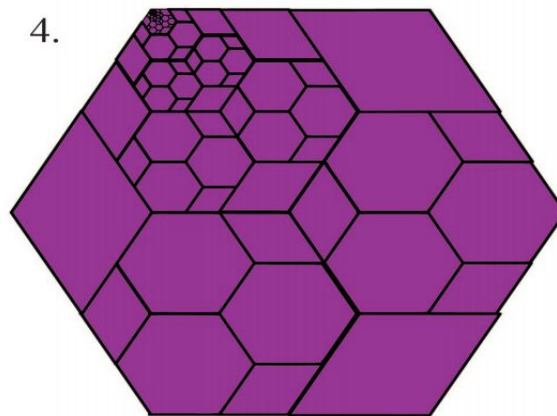
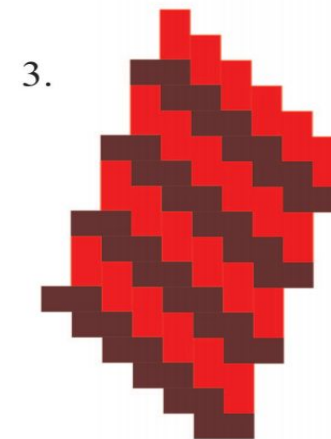
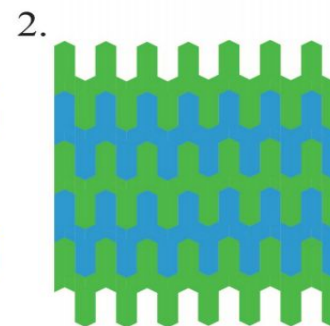
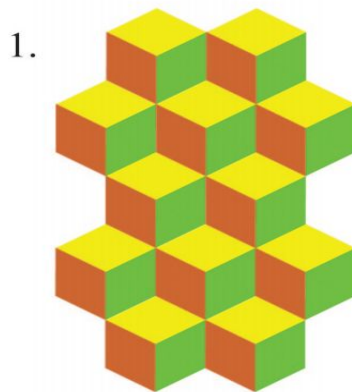
Non-example: D





# Tiling a Plane: Tessellations

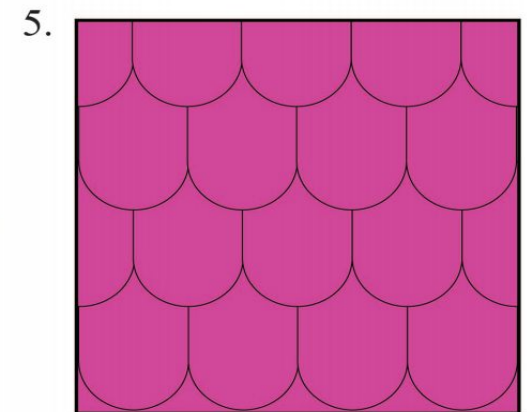
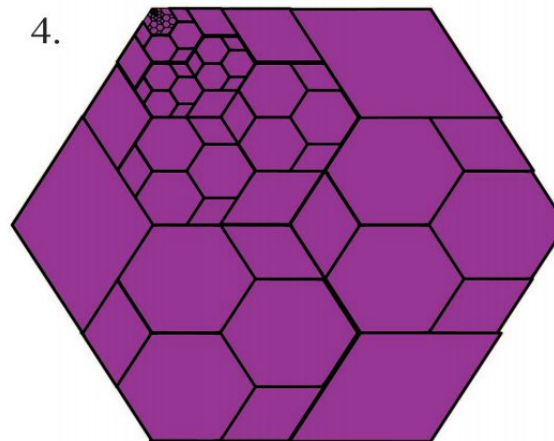
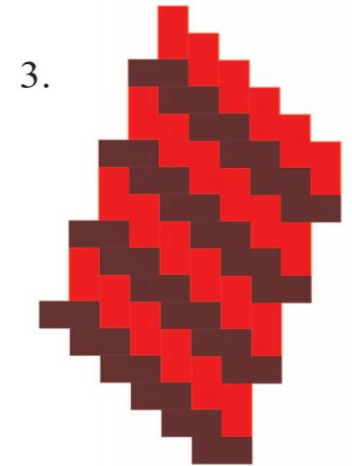
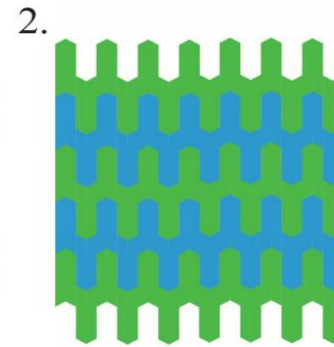
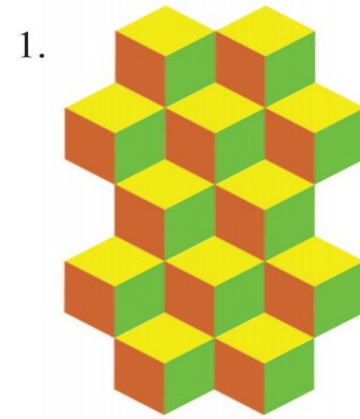
Examples or non-examples?



For each of the examples, answer the following questions:

a. How many *different* shapes (ignore the colors - they are misleading!) are used to tile the plane? You can think of these as buying white tiles and then paint them however you like.

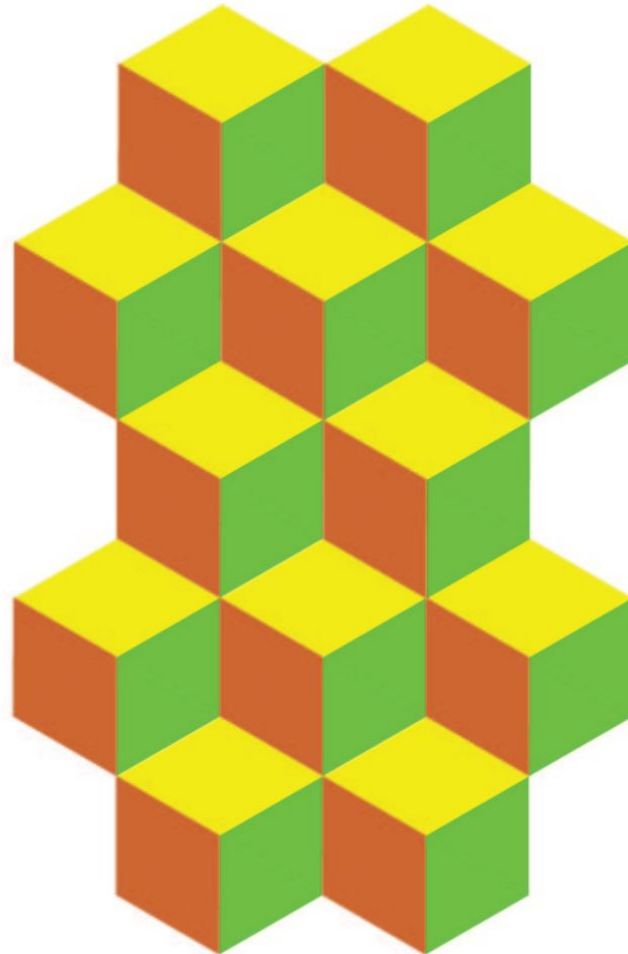
b. Describe the process of tiling using the tiles required!



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1.

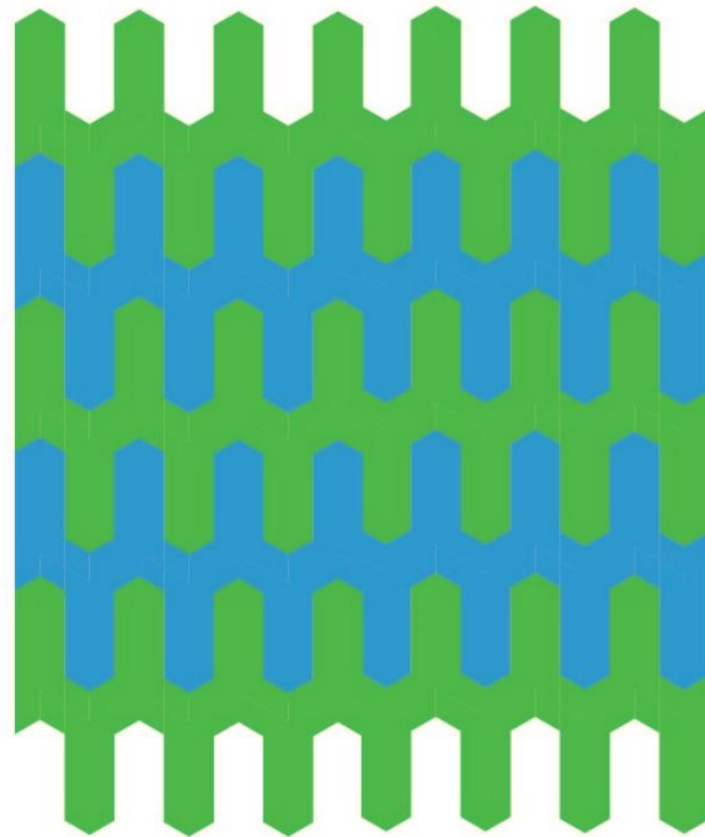




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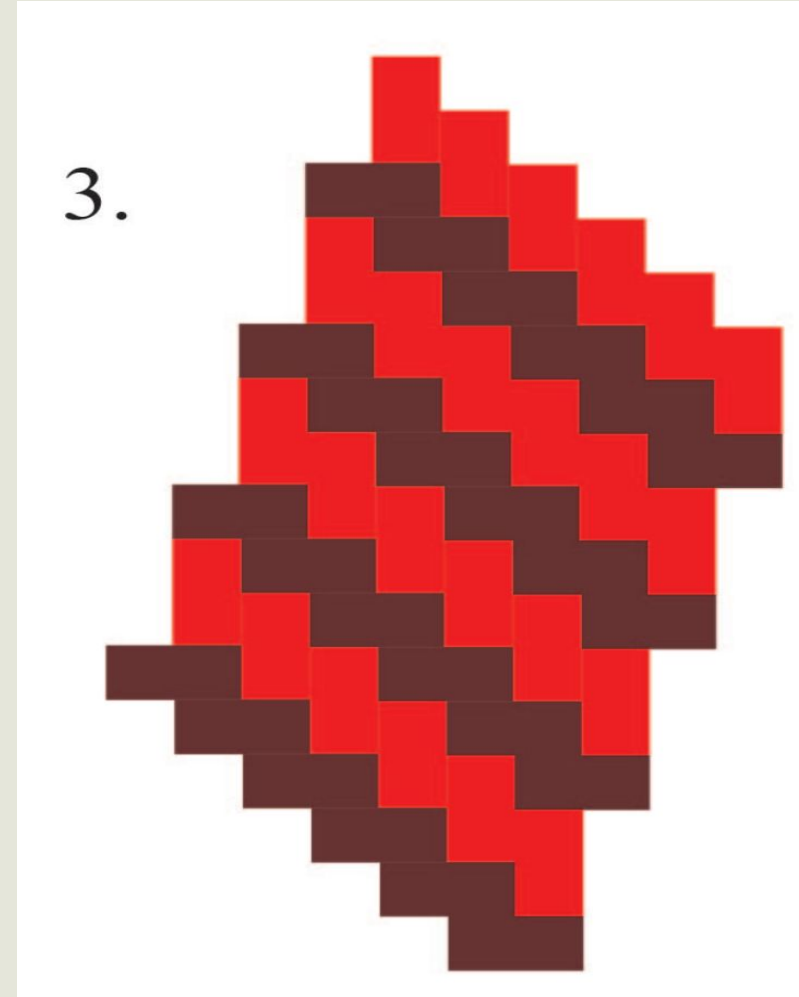
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2.



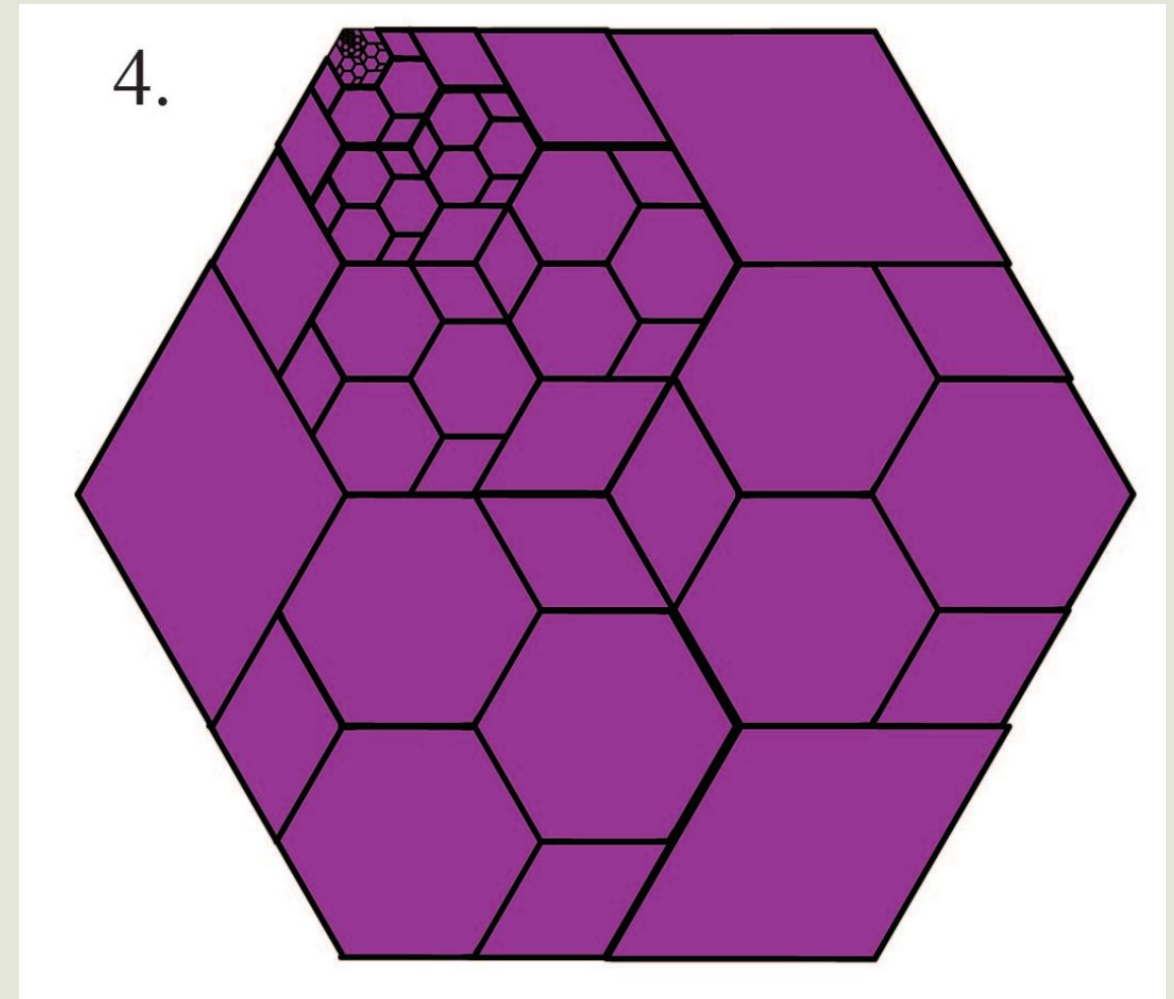
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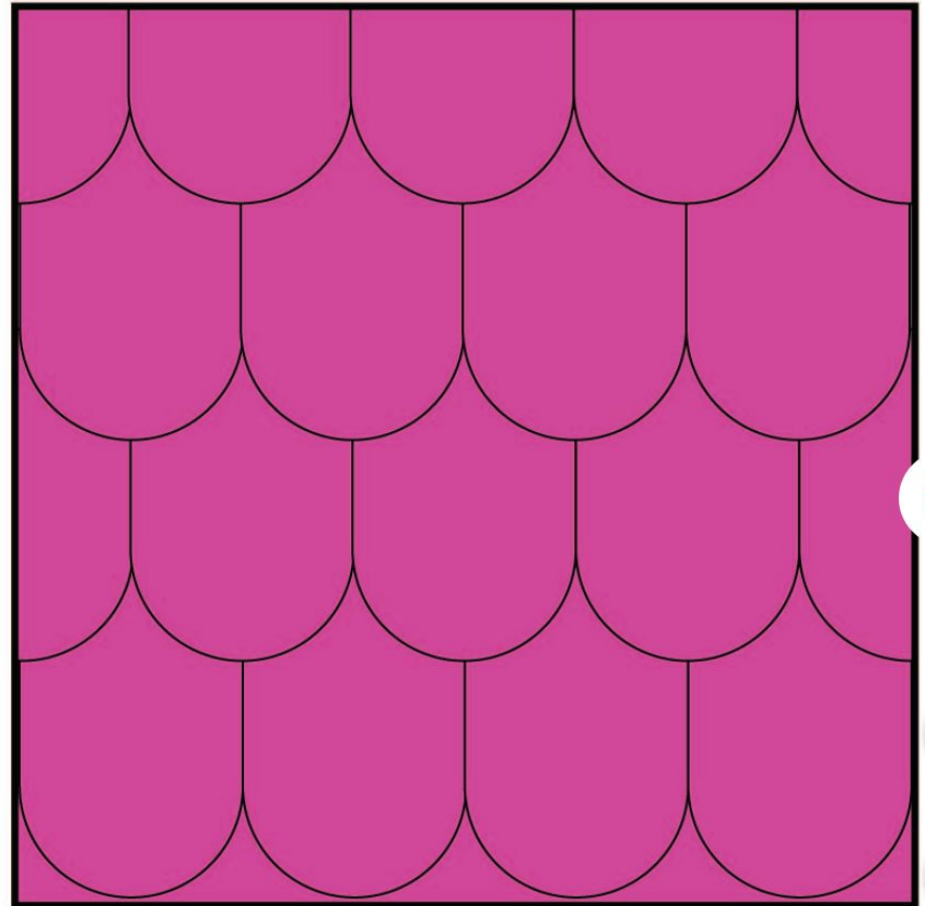




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- a. How many *different* shapes (ignore the colors - they are misleading!) are used to tile the plane? You can think of these as buying white tiles and then paint them however you like.
- b. Describe the process of tiling using the tiles required!

5.



Try to make your own shape that will tile the plane, using up to three different tiles!

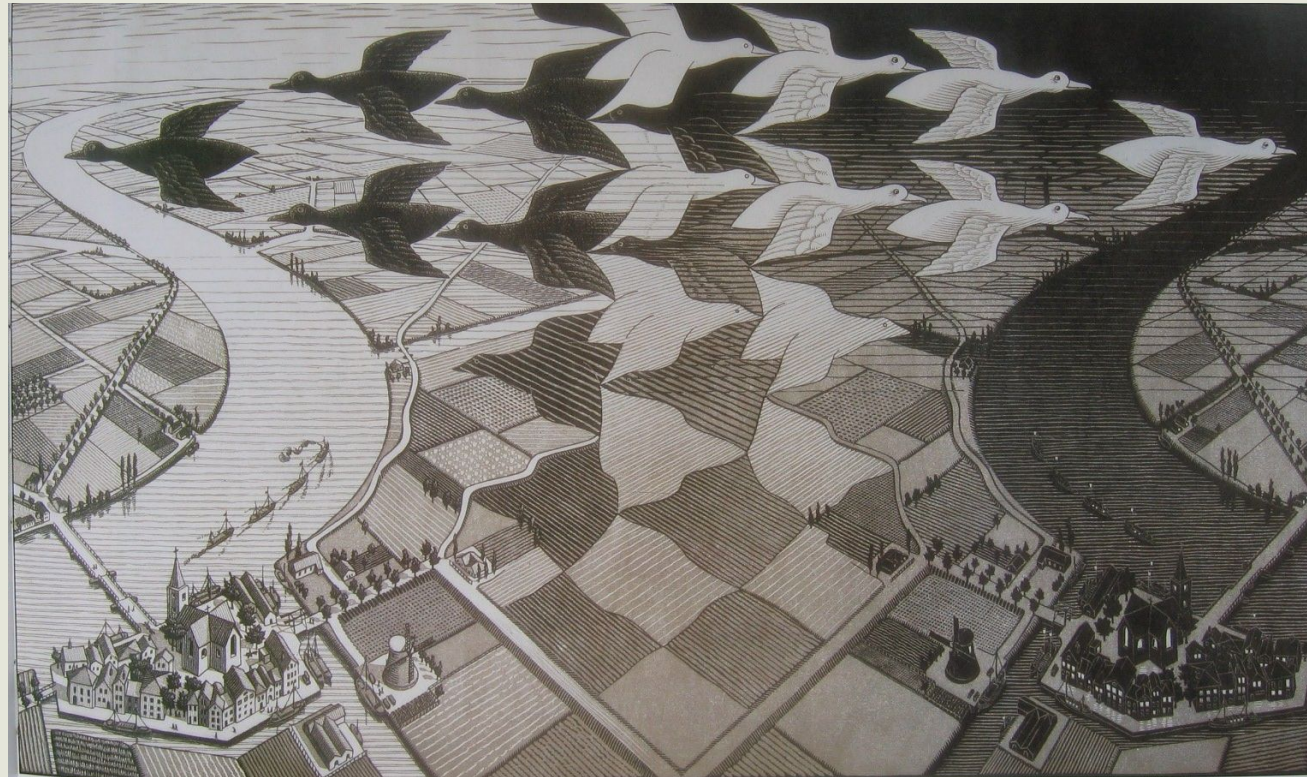
You can of course use any shape. But some strategies might help:

- Use polygons (shapes with straight sides rather than curved)
- Use symmetries (translation, rotation, reflection work best)



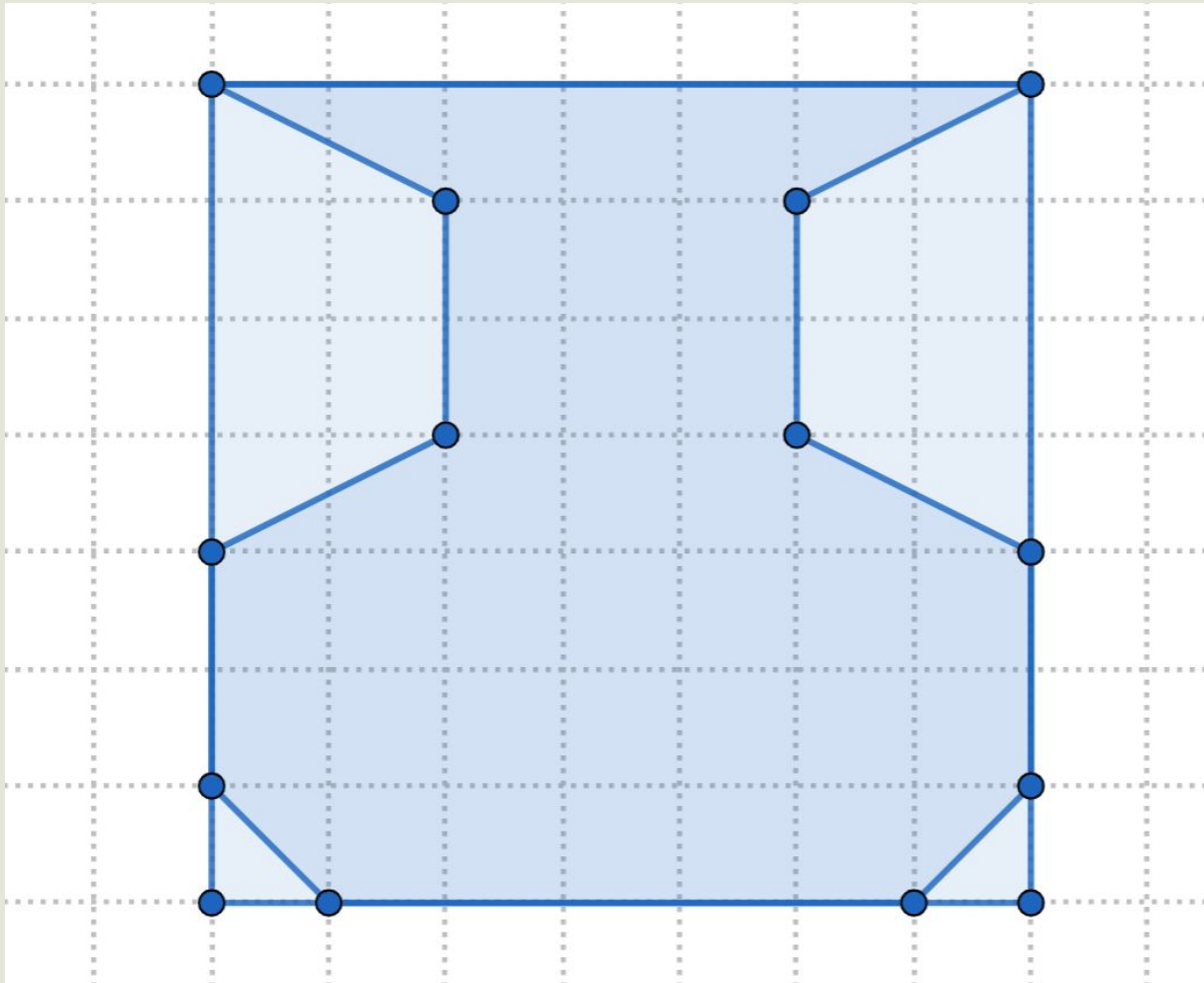
# Escher Tesselations

So here is one cool way to make tessellations, due to a Dutch artist M.C Escher. This technique has appeared on tapestries, wallpapers, floor tiles...





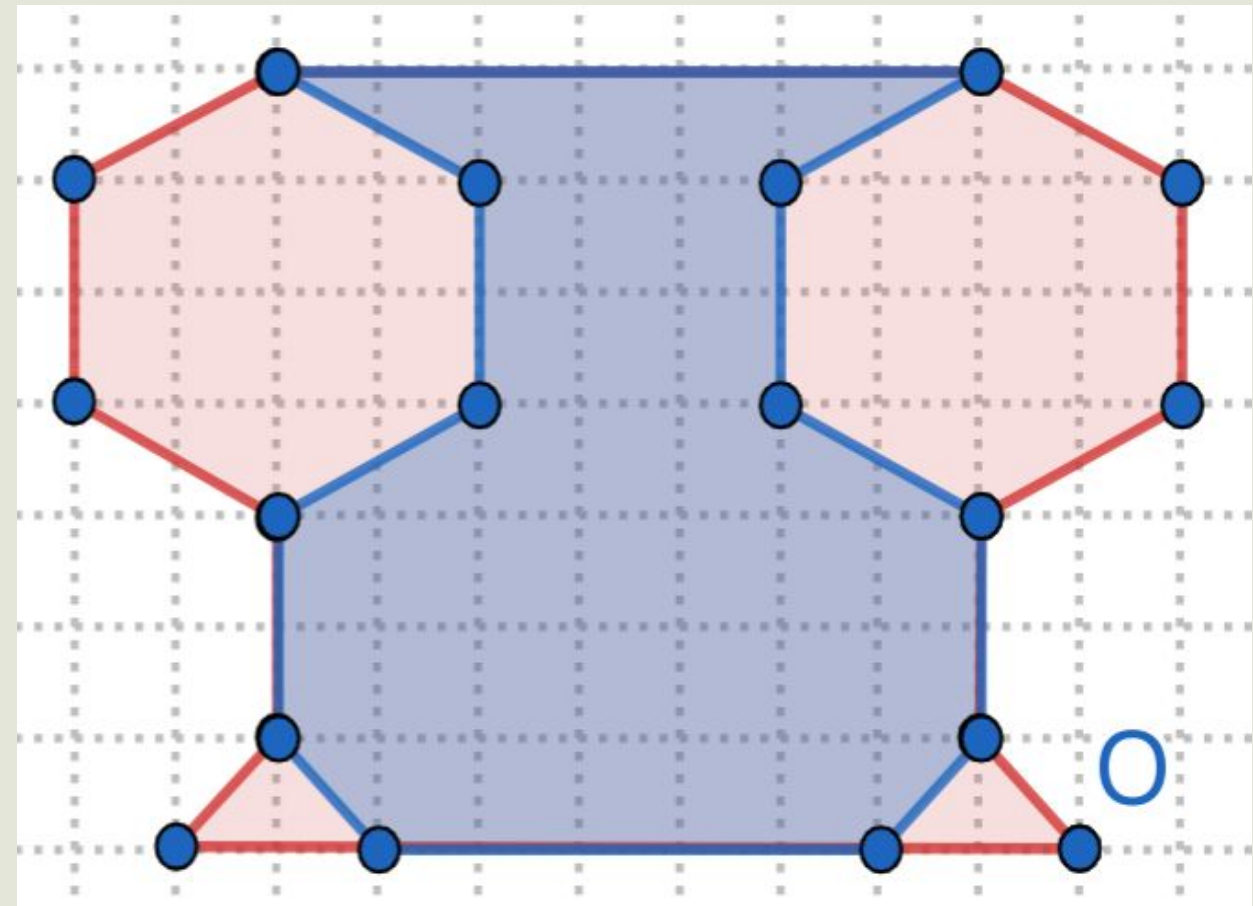
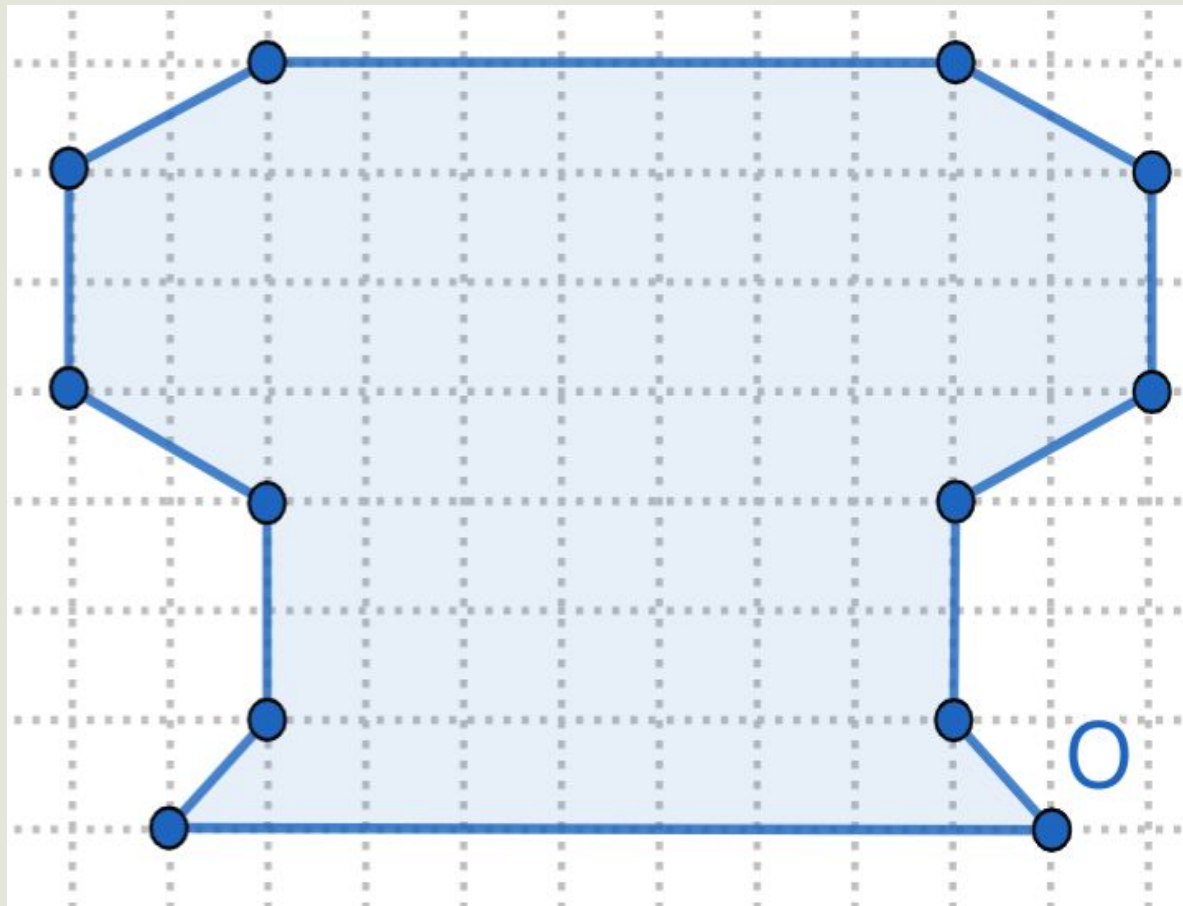
That looks very difficult! So we will learn the basic principles and you can explore at home!







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What does it look like when you put them together?

I'll draw on the board! :)

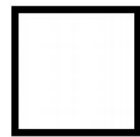
You should play with this idea at home with colored grid paper!

# Which special shapes can tile a plane and WHY?

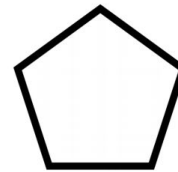
We will focus on regular polygons! What is a regular polygon?



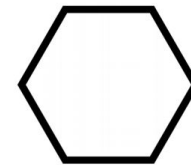
Triangle



Square



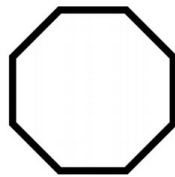
Pentagon



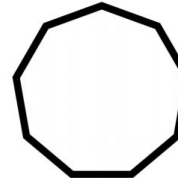
Hexagon



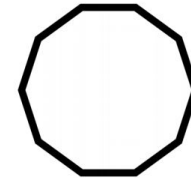
Heptagon



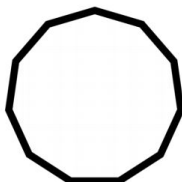
Octagon



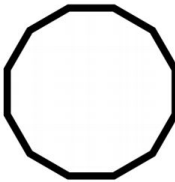
Enneagon



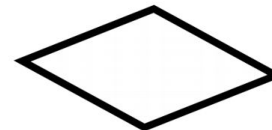
Decagon



Hendecagon



Dodecagon



Rhombus



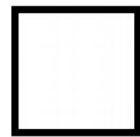
Trapezoid

# Which special shapes can tile a plane and WHY?

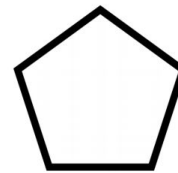
Which regular polygons can tile the plane by itself? Explore and practice!



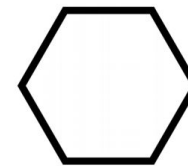
Triangle



Square



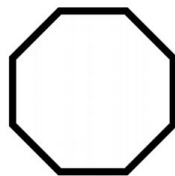
Pentagon



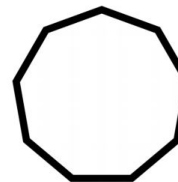
Hexagon



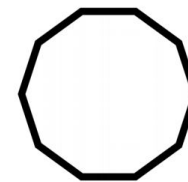
Heptagon



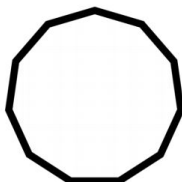
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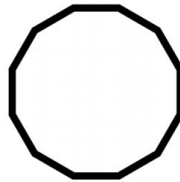
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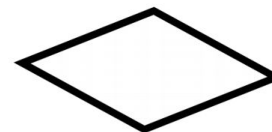
Decagon



Hendecagon



Dodecagon



Rhombus



Trapezoid



# Sum of all interior angles of a regular n-gon?

Number of Angles	Total Degrees	Measurement of Each Angle
3	180	60
4		
5		
6		
8		
n		

# Which regular polygons can tile the plane by itself?

Number of Angles	Measurement of Each Angle	Can Tile
3	60	YES
4		
5		
6		
8		
n		--

## What about combinations of regular polygons?

1. We want to remodel 1015 Evans and retile the floor using a combination of two types of regular polygons from selection of 3-gon, 4-gon, 6-gon or 8-gon. Which combinations are possible?
2. What if we want to use three different types?



See you next  
week on Tiling a  
Board!

# Board Tiling, Chocolate Breaking with a Hint of Fibonacci

Part I

By Harry Main-Luu