Board Tiling, Chocolate Breaking with a Hint of Fibonacci

Part II

By Harry Main-Luu

General Overview

Part 1: Tiling a Plane

Part 2: Tiling a Board

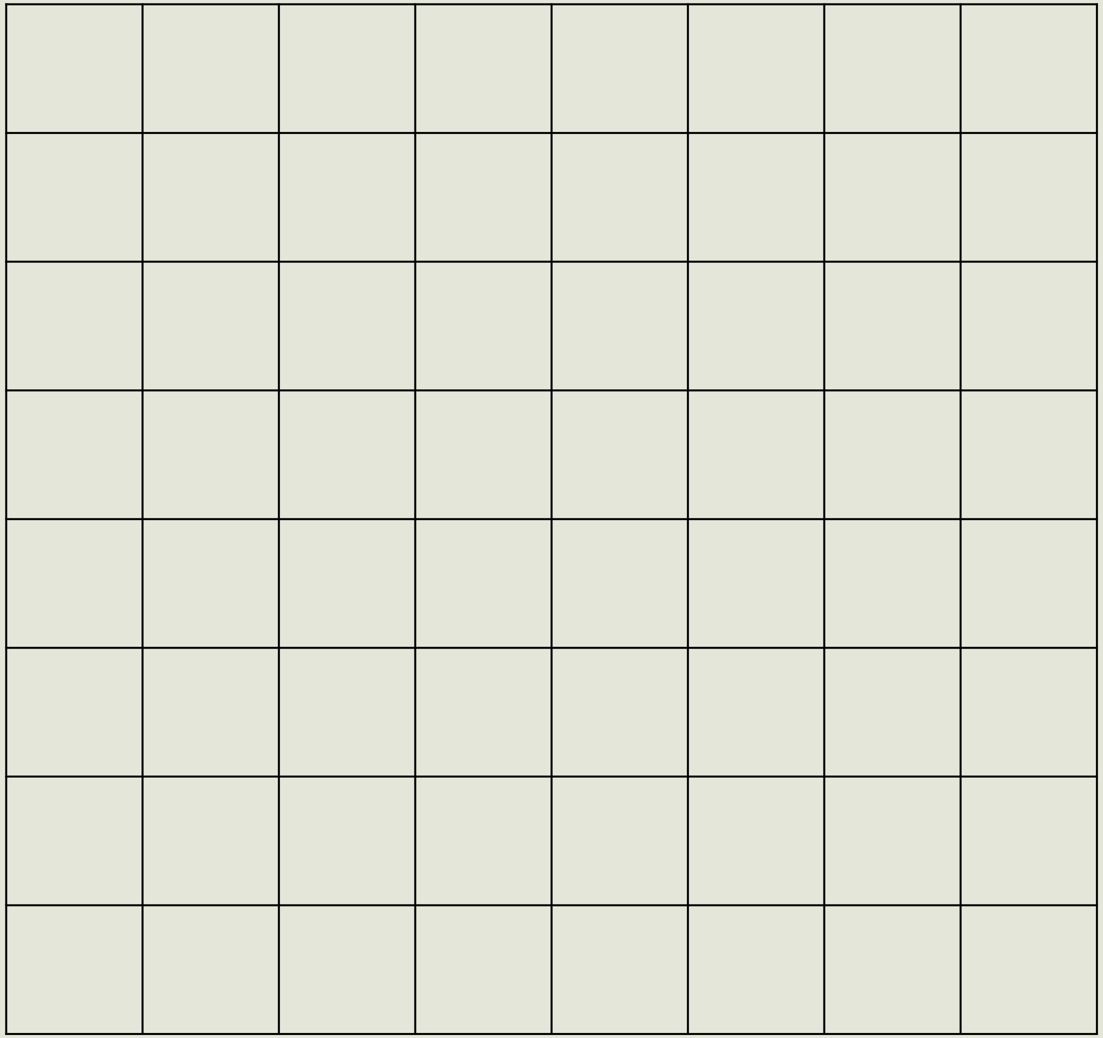
Part 3: Breaking and Sharing Chocolate

Overarching question for today’s sessions:

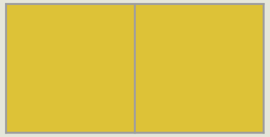
- Given a finite board, is it POSSIBLE to tile it with certain shapes, such as dominoes, triominoes, T-shapes...?
- Next week: We will talk about how many ways we can do it if it’s possible!

Some materials from this session are taken from the BMC Book v1, Session 10; National University of Vietnam, HCMC High School for the Gifted contest, and other open sources.

Tiling a Chessboard (8x8 Grid)



Can you tile this board with dominoes?



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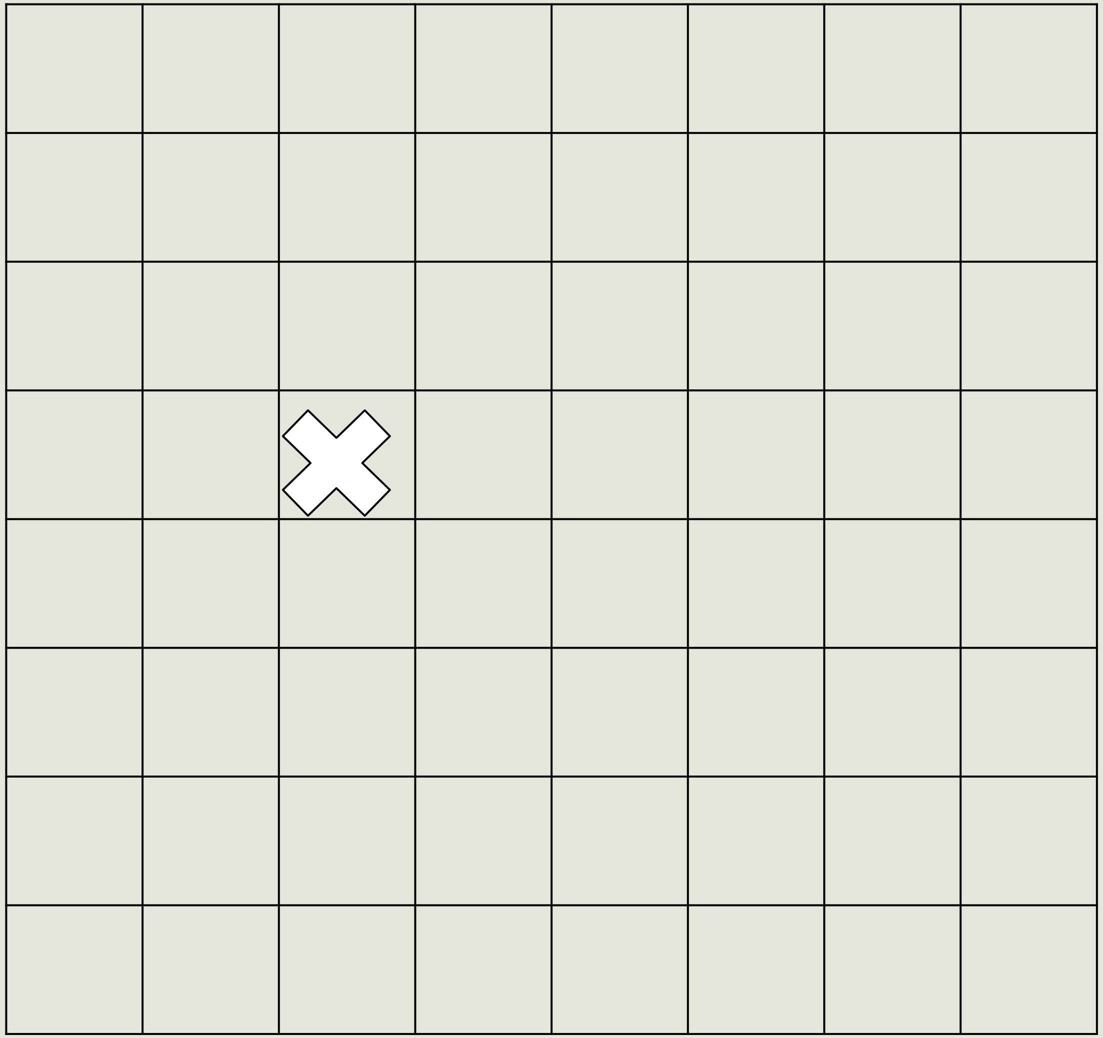
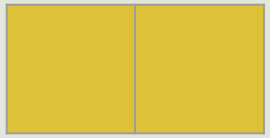
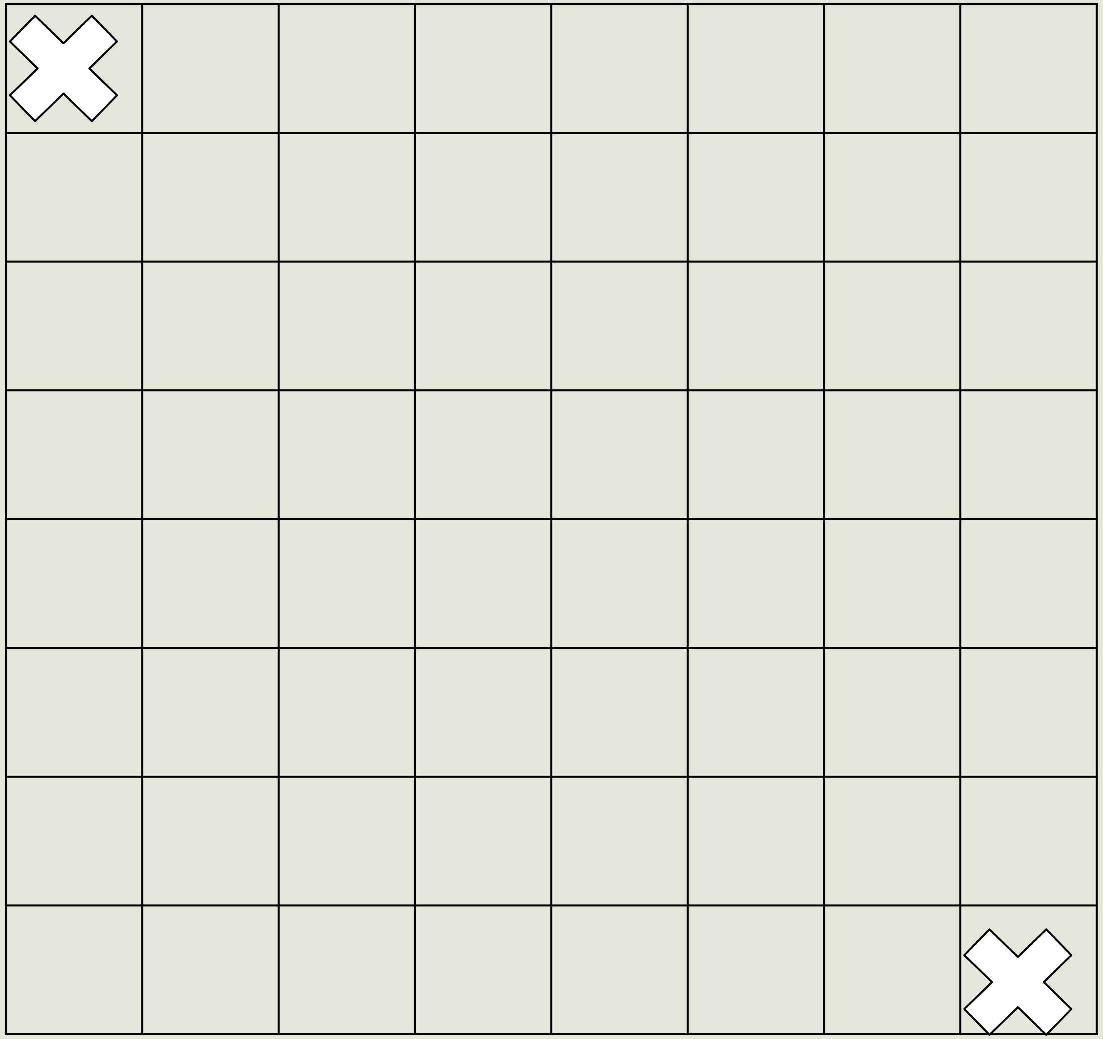
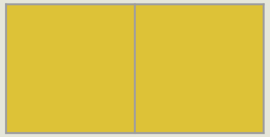


What if we remove one square? Can you still tile this board with dominoes?



Experiment: With your partner, remove two random squares on the 8x8 grid. Can you tile the resulting board?





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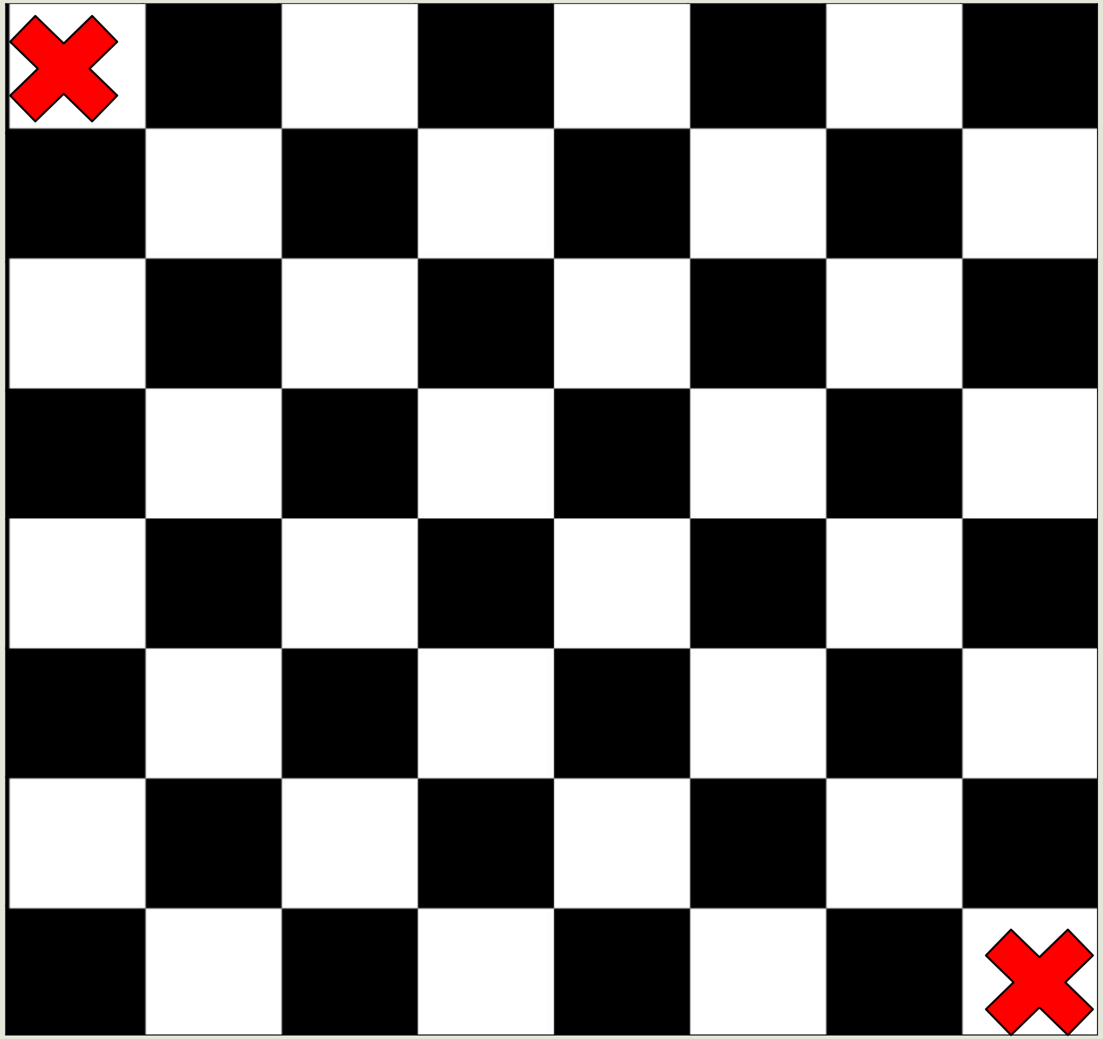

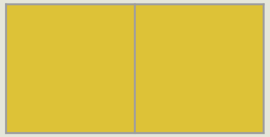

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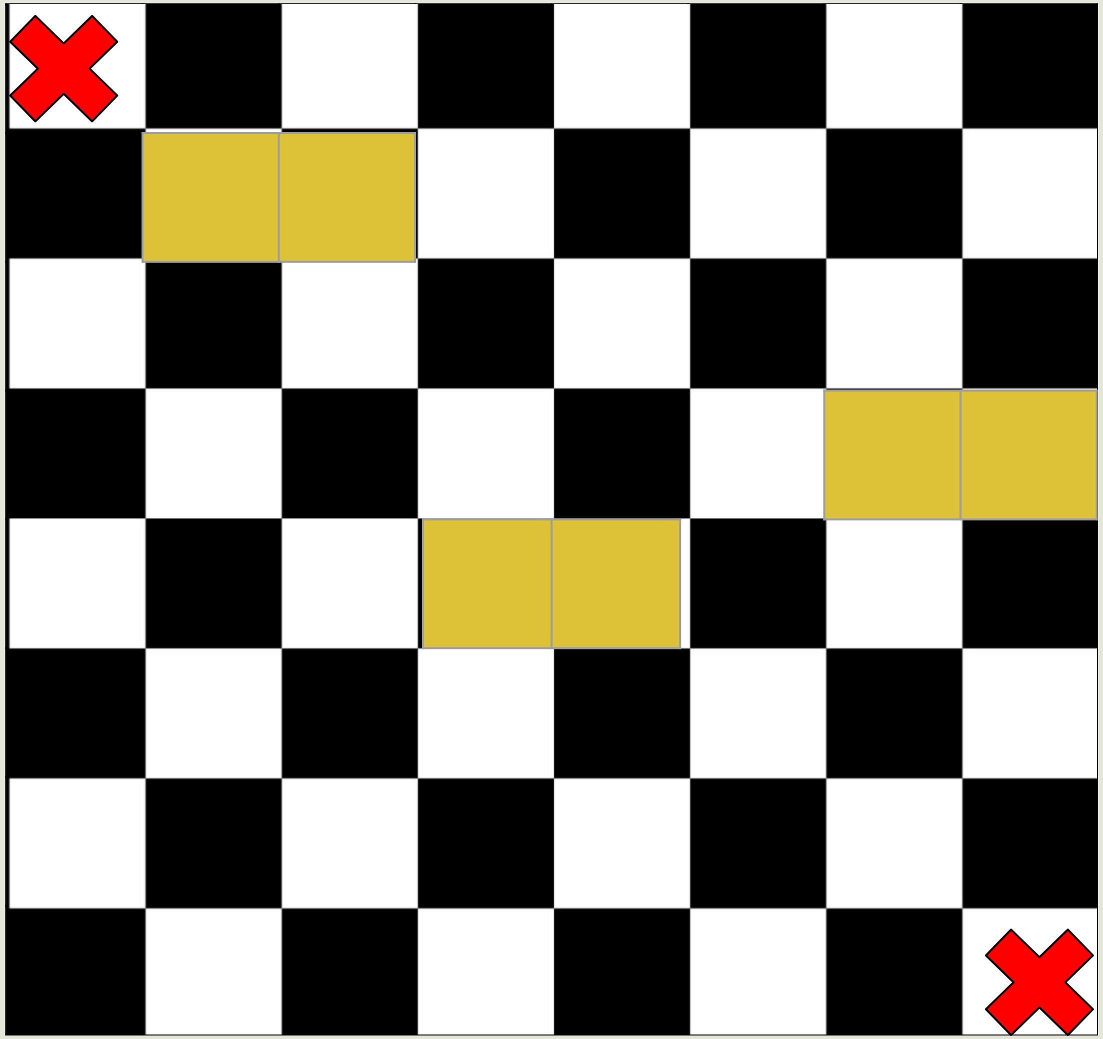

What happens when we place a domino on the chessboard?

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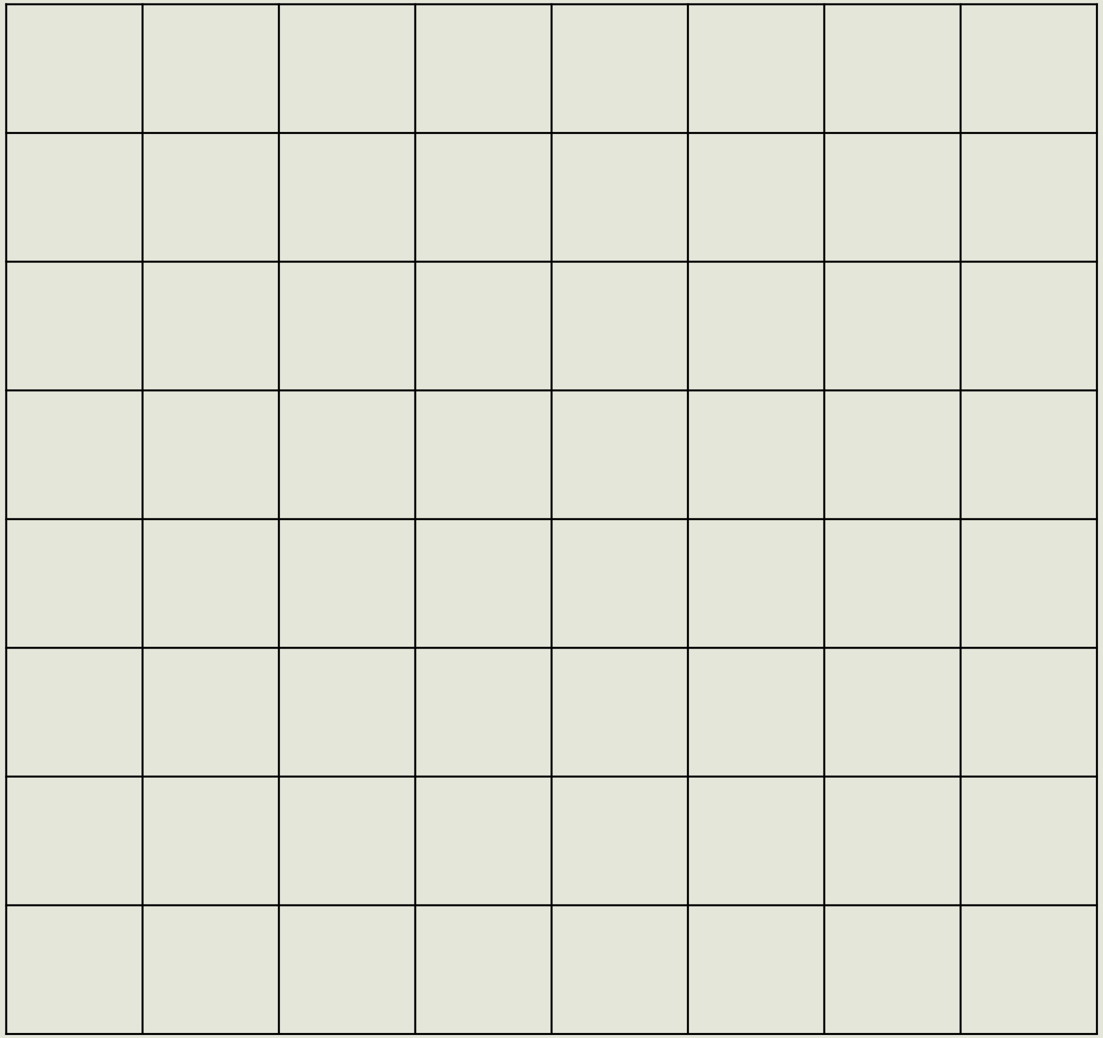
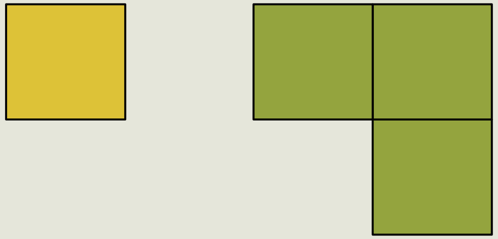
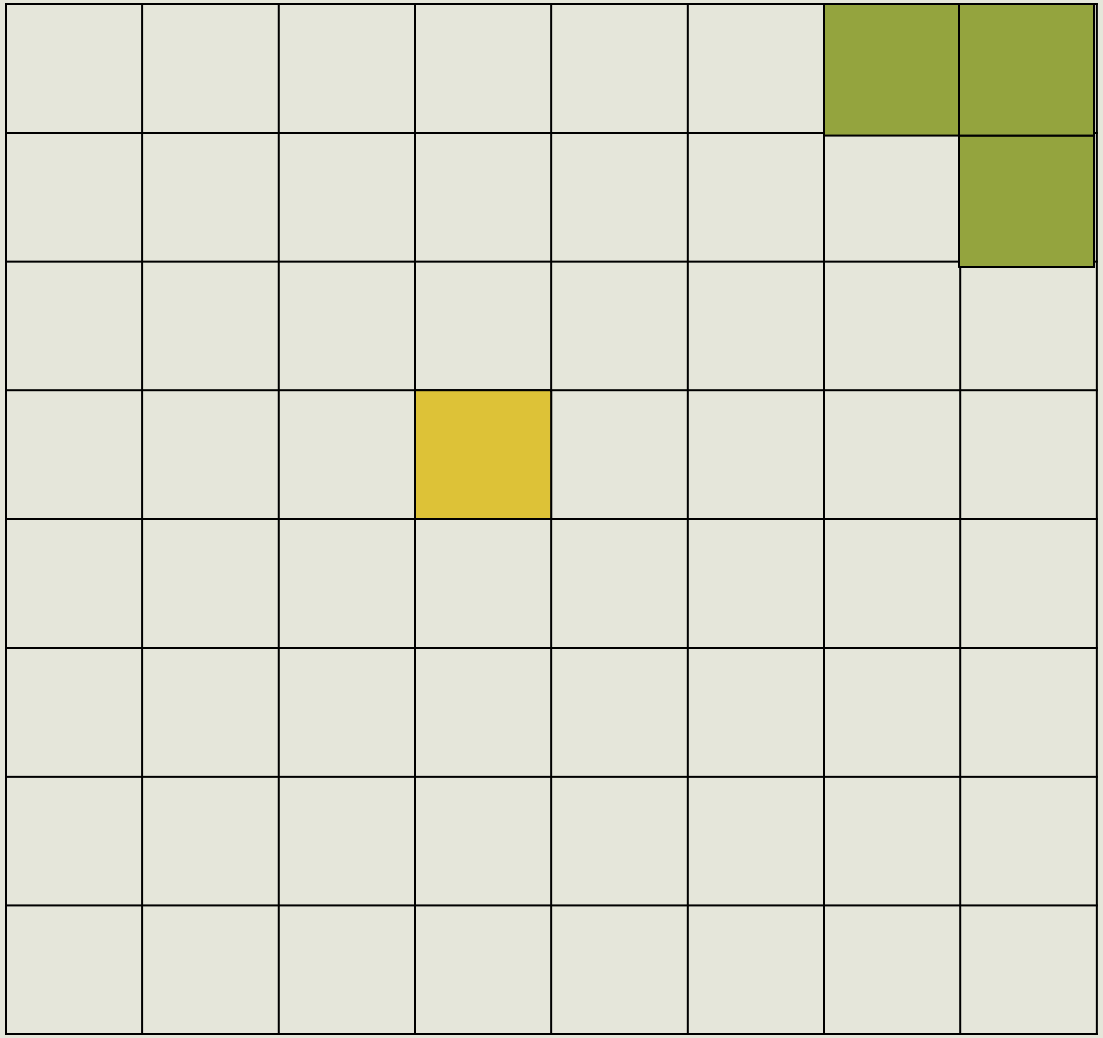


Can you tile this board with one special square and a lot of the L shape tiles?



Experiment: Put the special square anywhere, then try to tile the rest with the L tiles.



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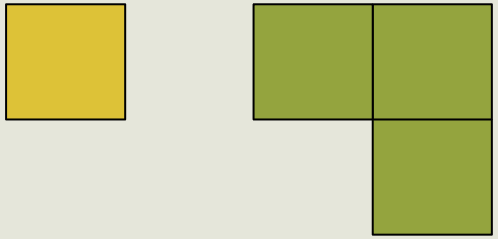
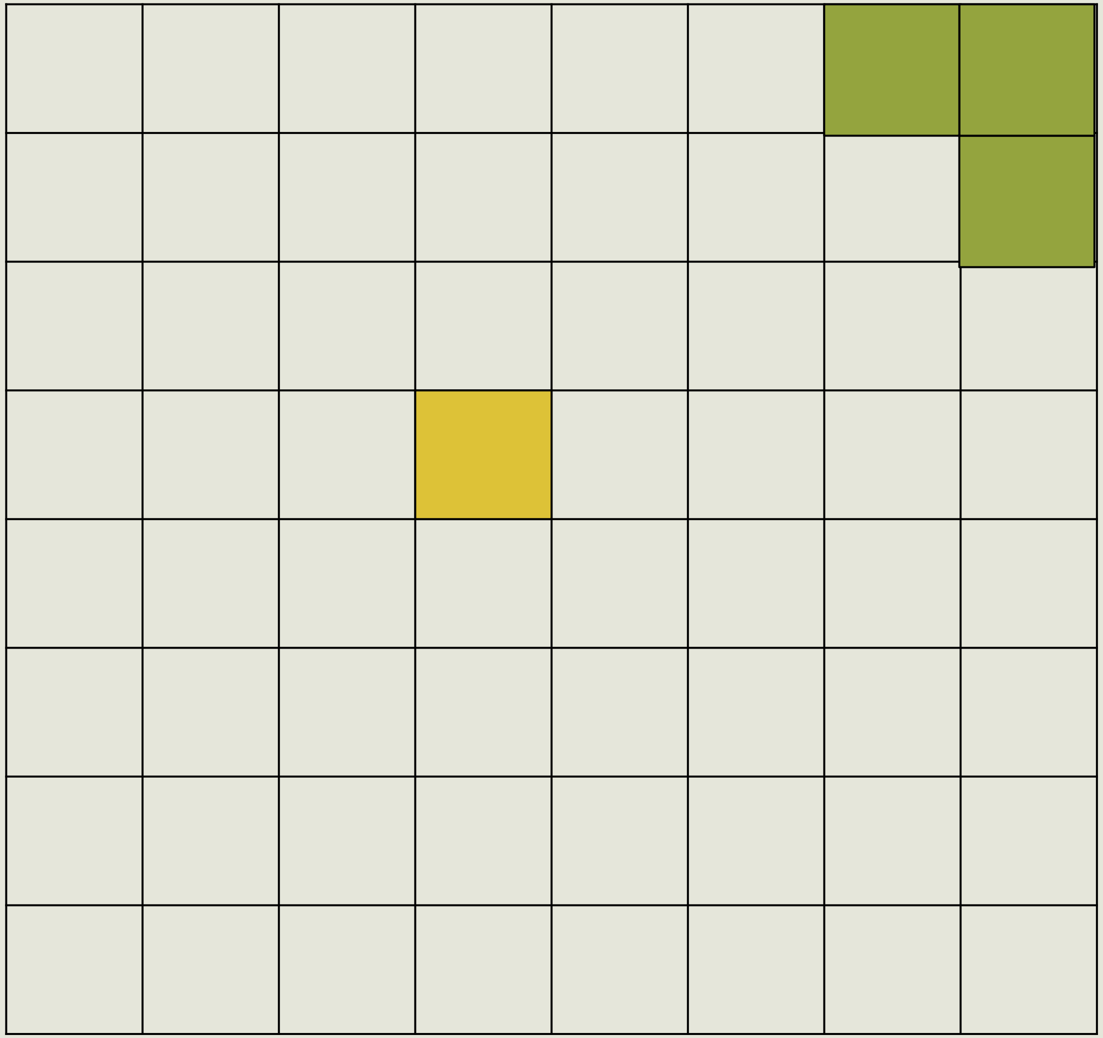
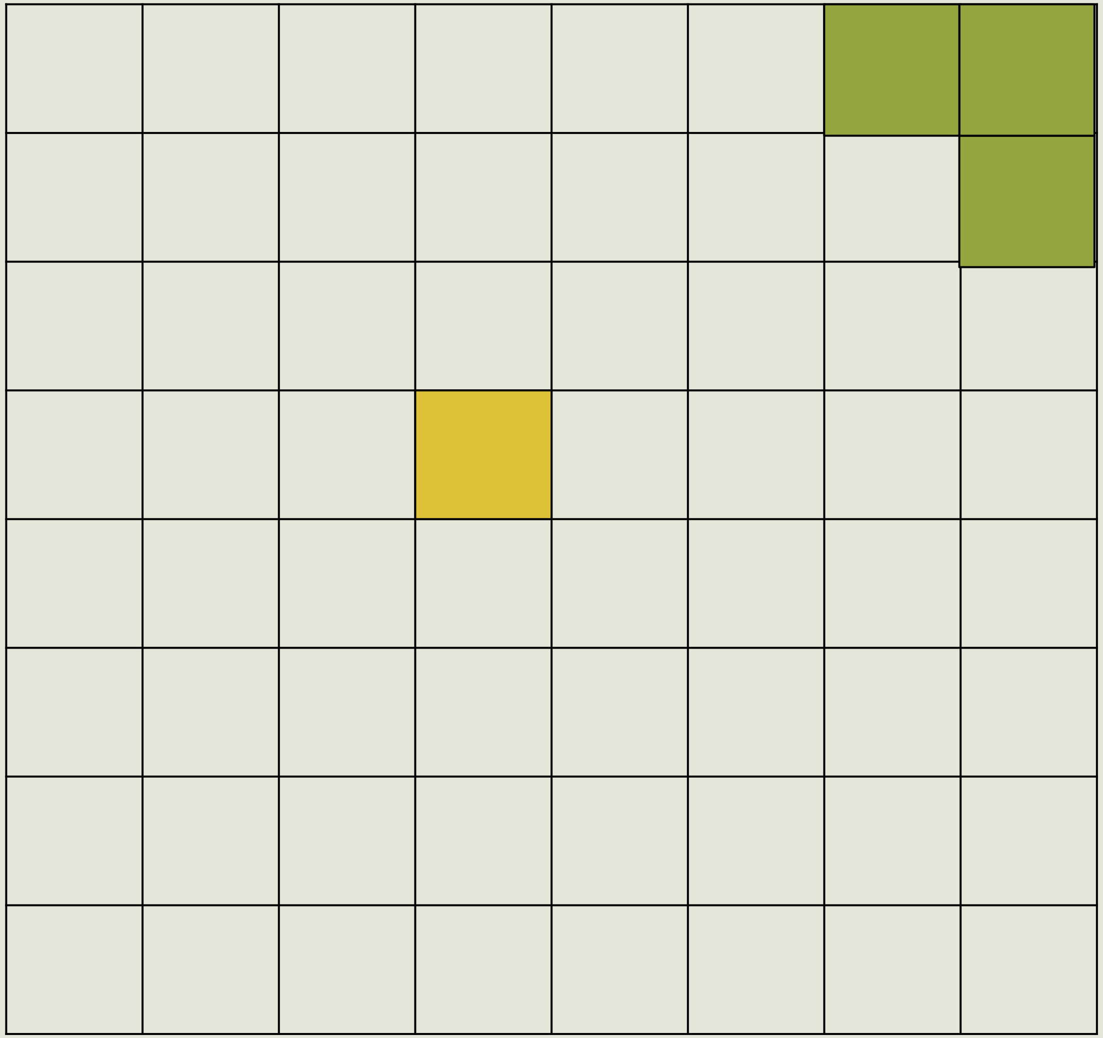


How do we prove that it is ALWAYS possible to tile the 8x8 grid regardless of where the yellow square is?



Perhaps proof by exhaustion?

It is always possible to tile a $2^k \times 2^k$ board with one special square and many L shape tiles, for any natural k.

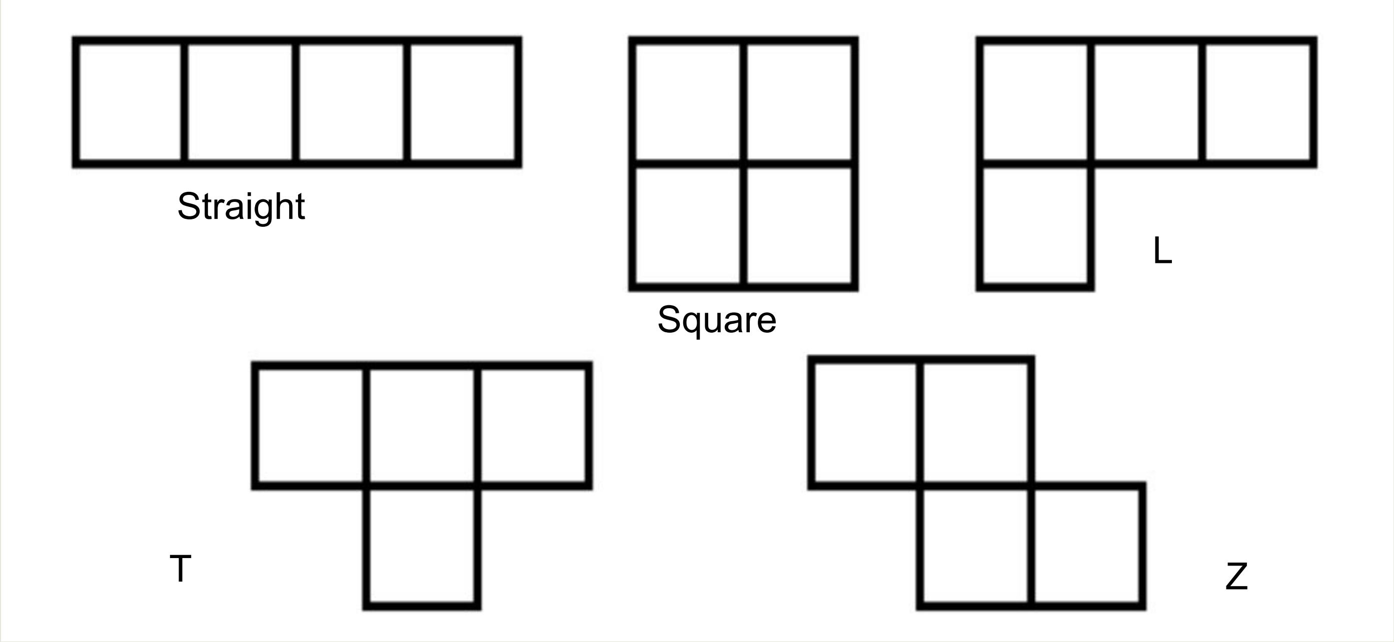


How do we show that something is possible for infinitely many options?!

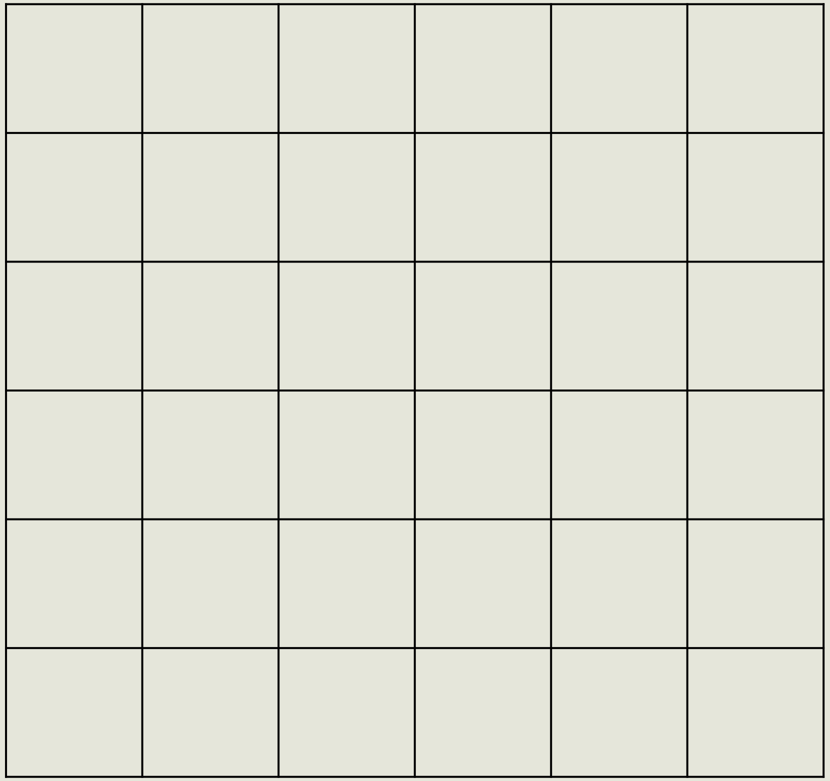
Tiling a chessboard

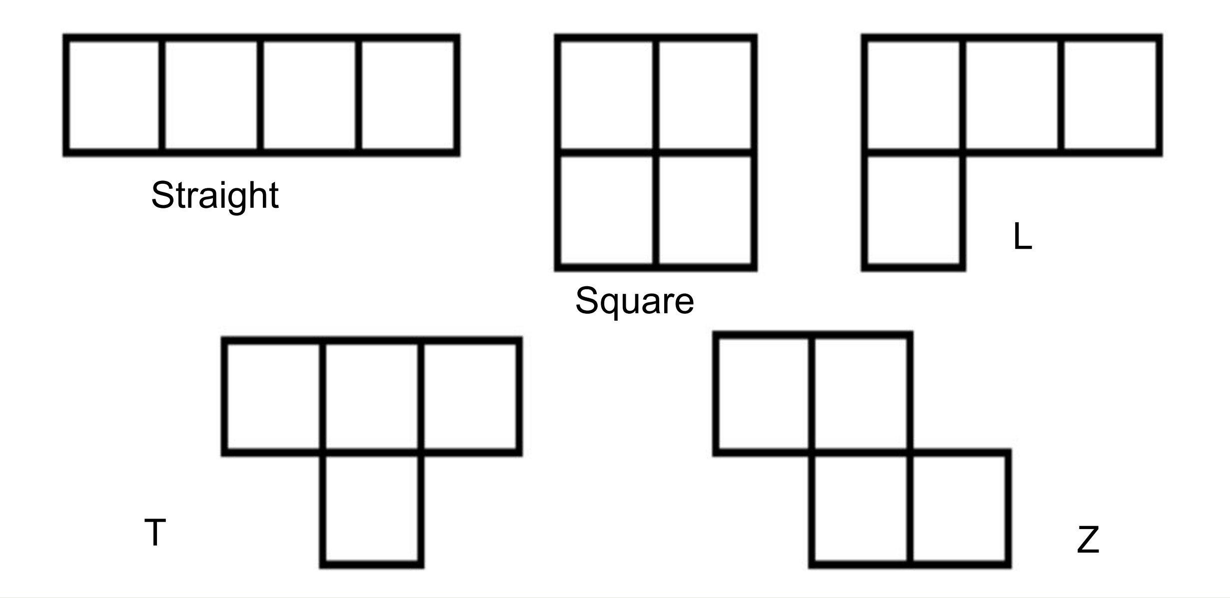
1. Show that it is POSSIBLE to tile a chessboard with the first four types of tetrominoes.

2. Which ones of these can tile a 6x6 board?

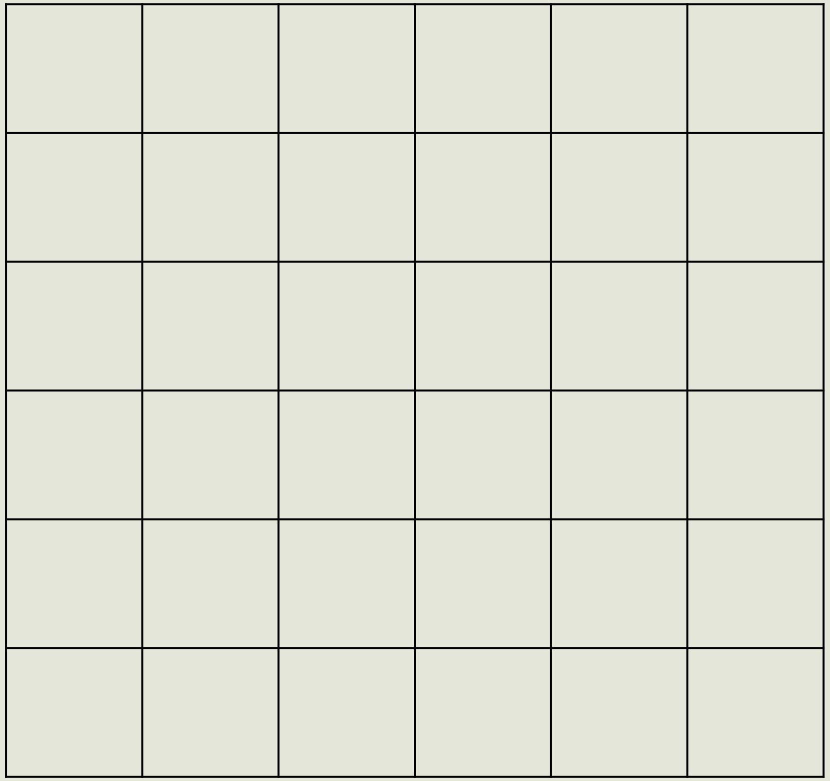


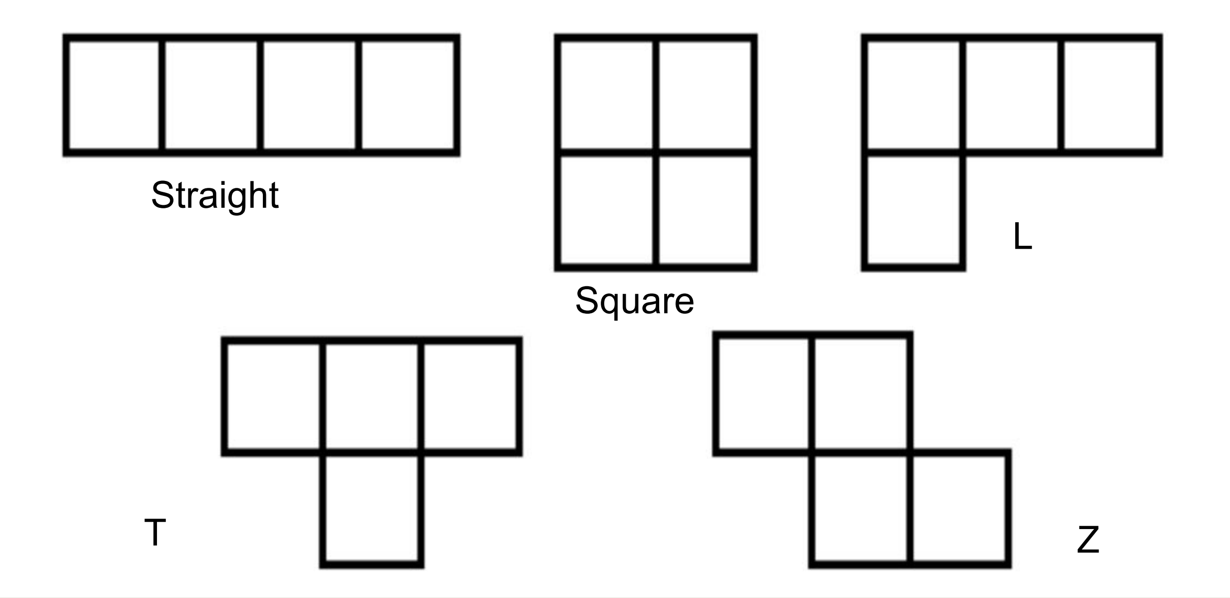
Tiling a 6x6 board with the Square's





Tiling a 6x6 board with the straight's





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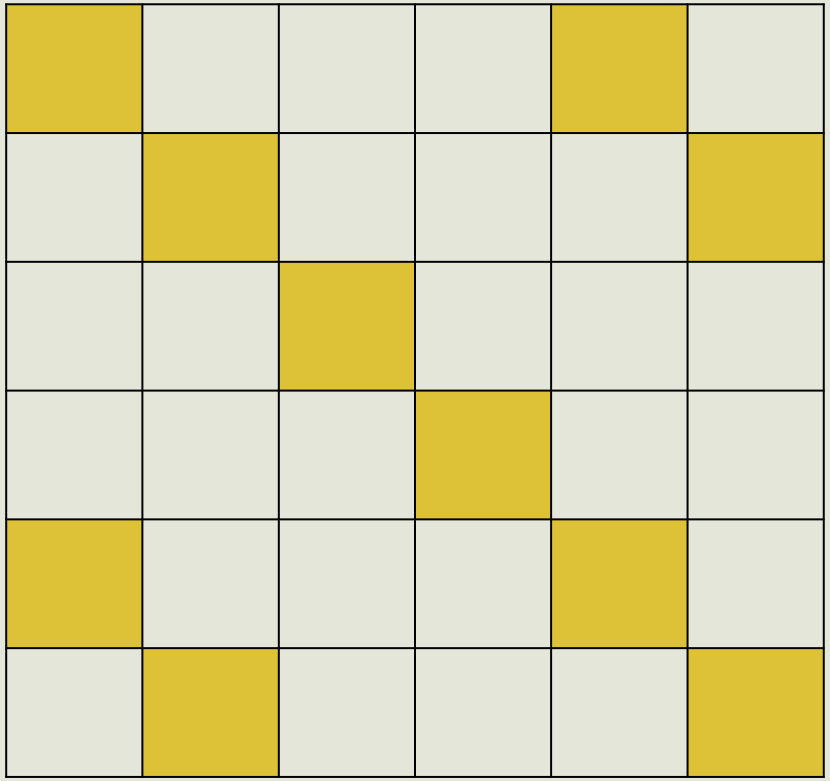
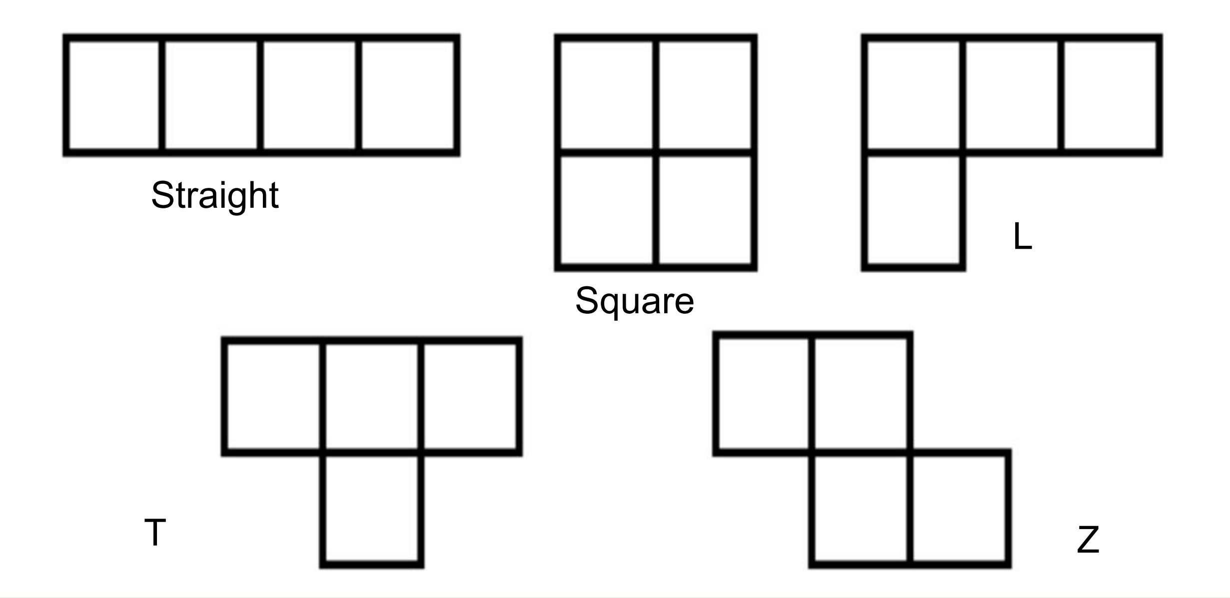
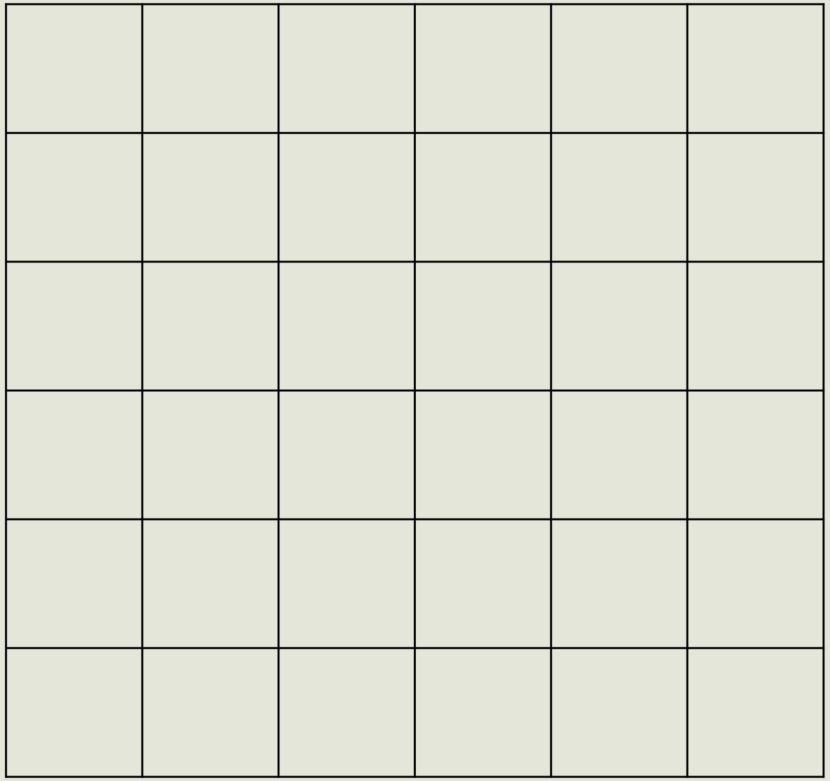
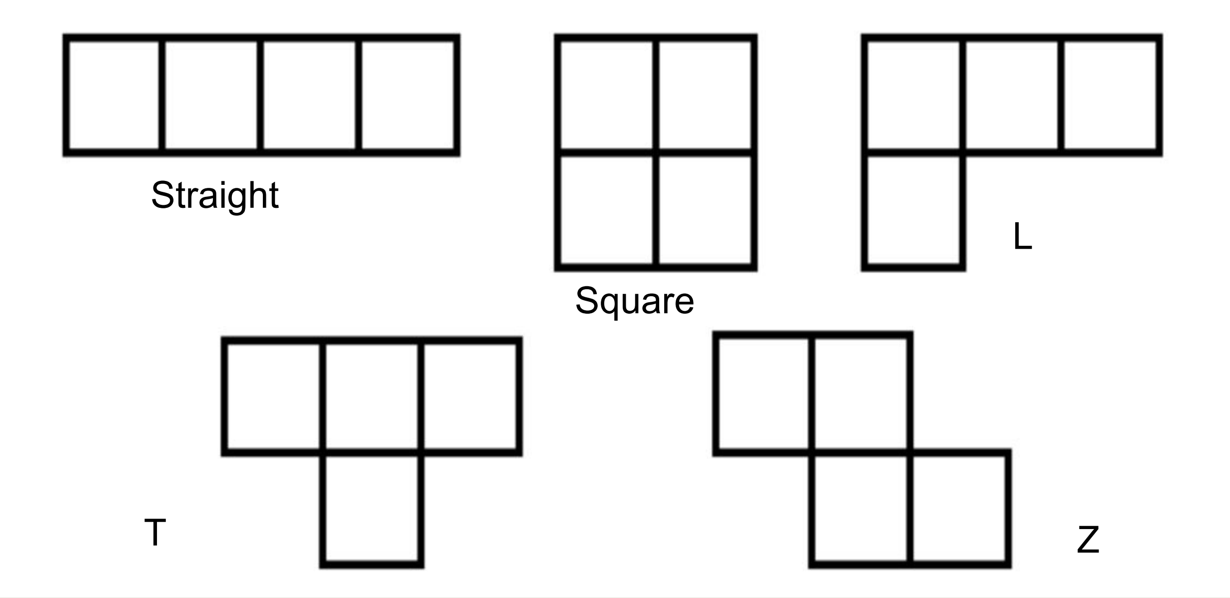




Tiling a 6x6 board with the Z's





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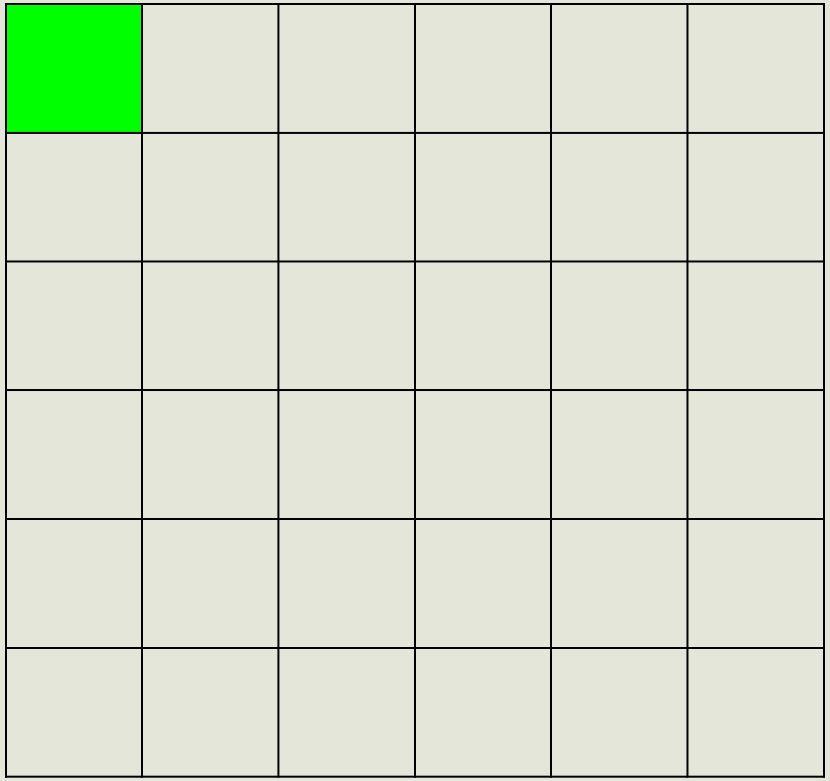
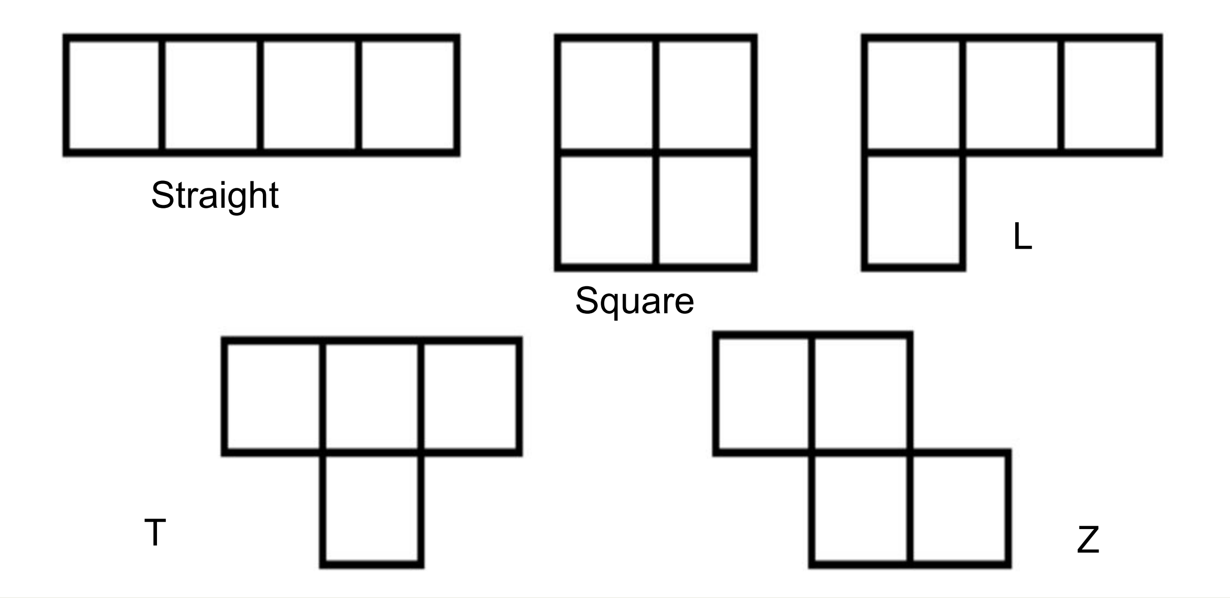
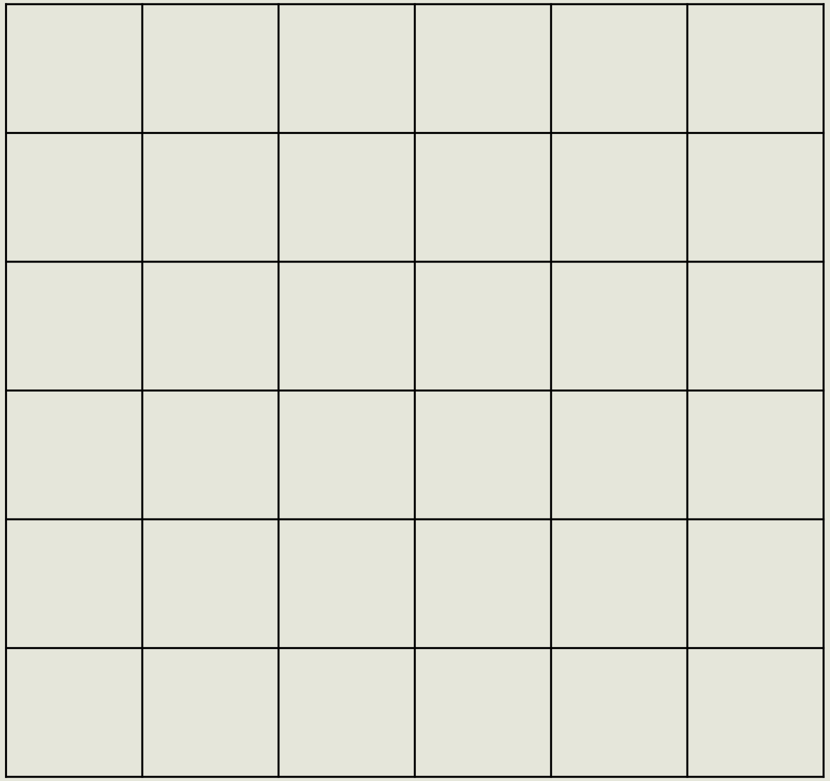
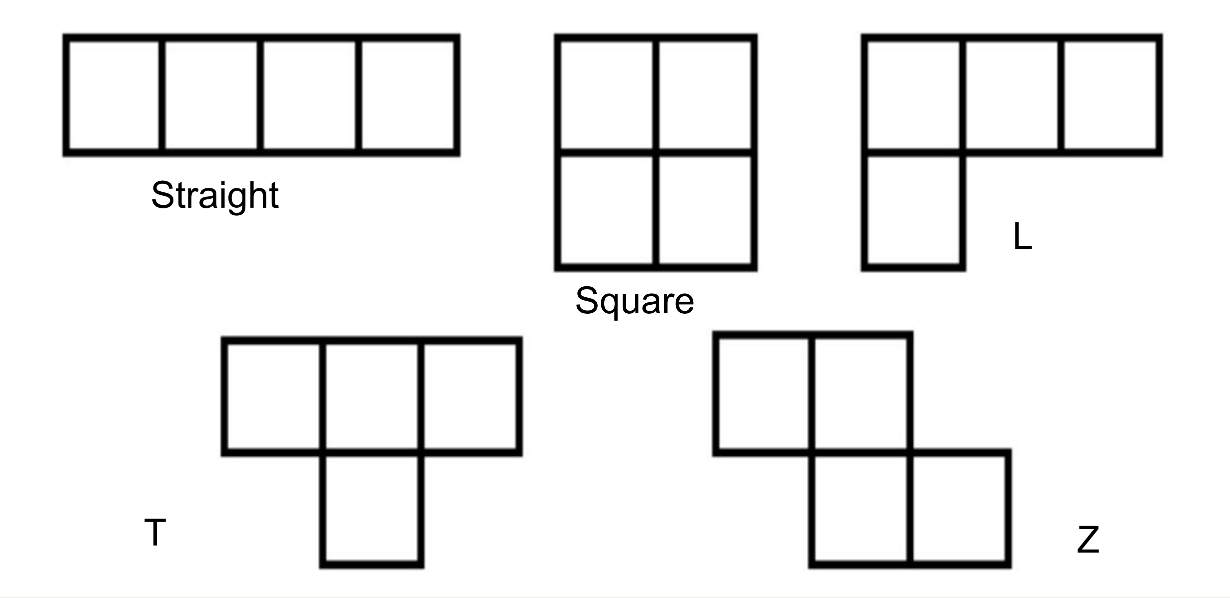




Tiling a 6x6 board with the T's





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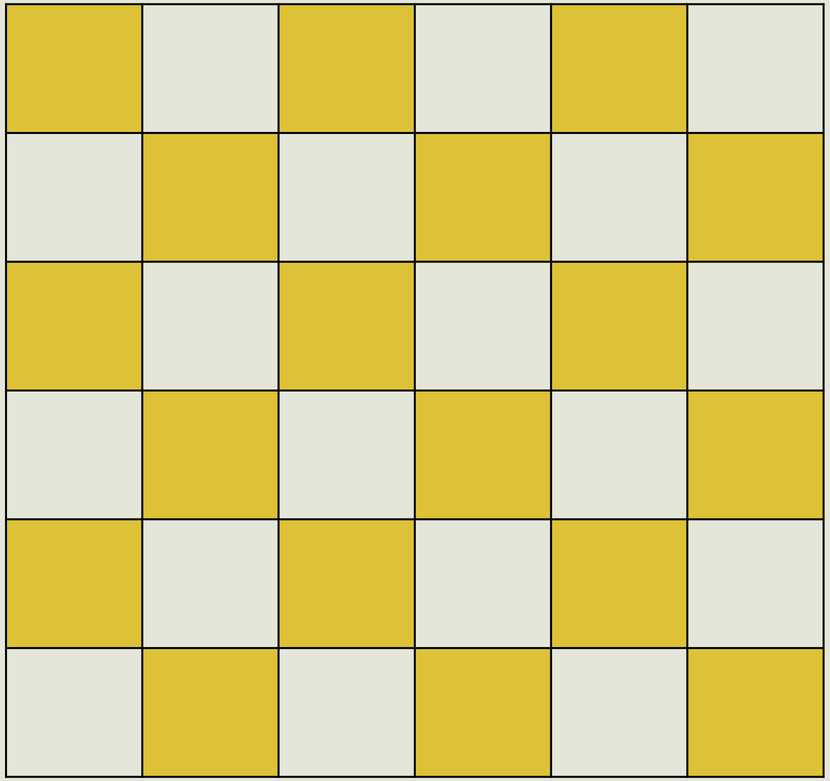
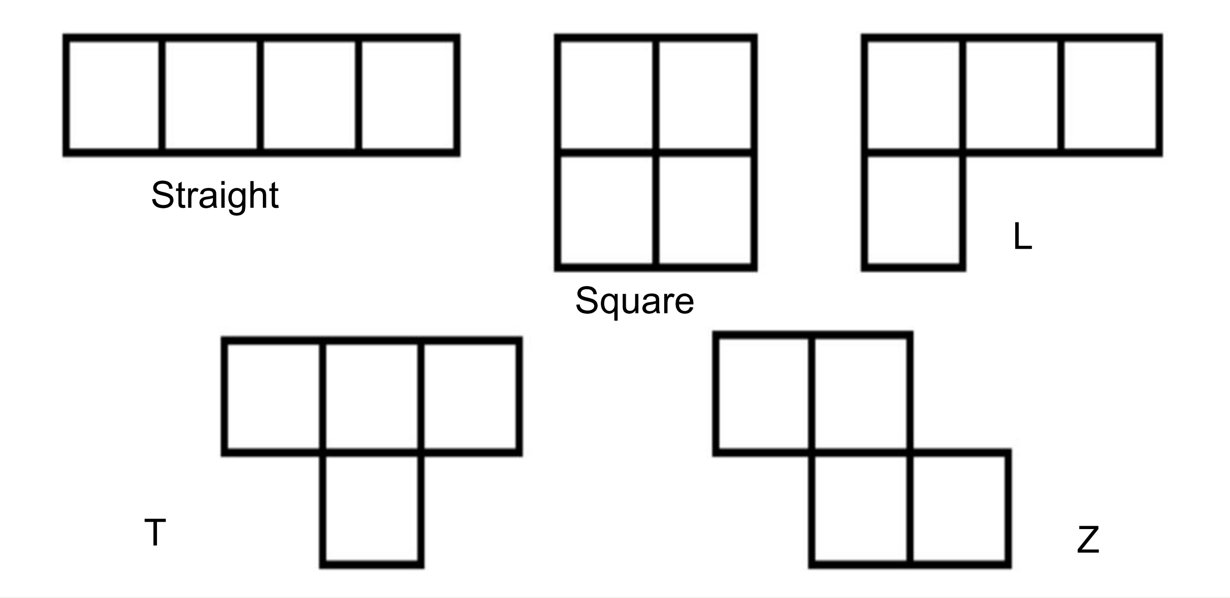
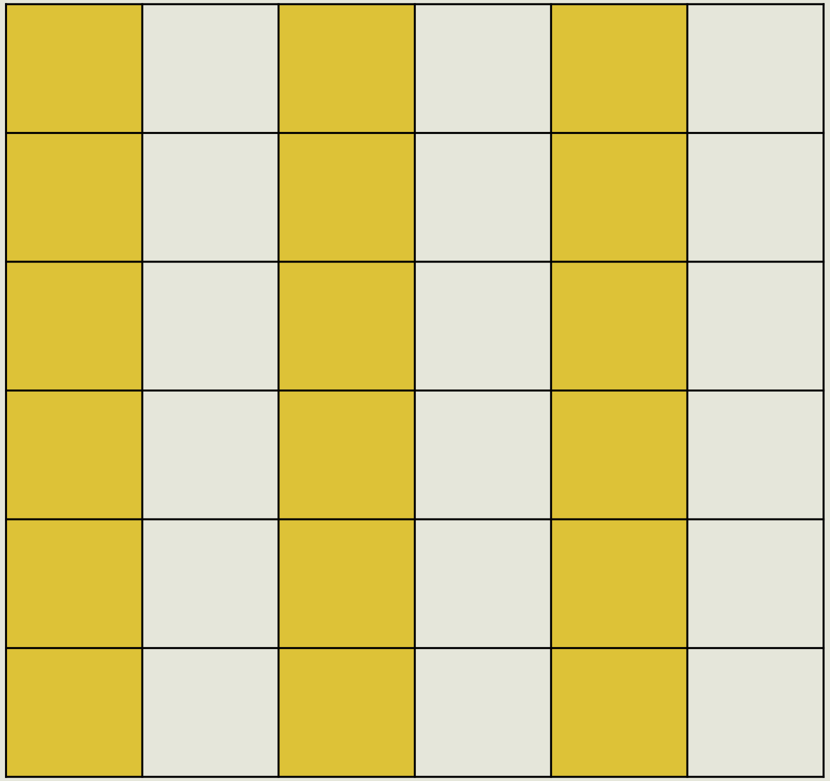
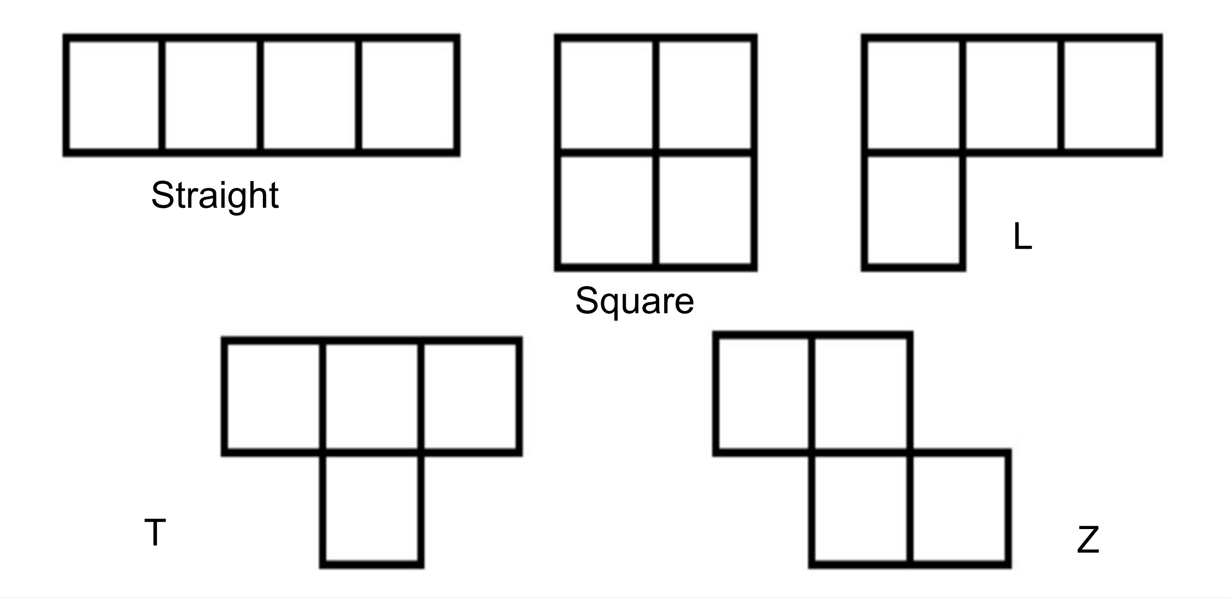




Tiling a 6x6 board with the L's





Preamble for next week (IF we have time):

1. How do we solve a quadratic equation?
2. What is the square root of a number?
3. How do add two fractions? What about adding two fractions with variables instead of numbers?
4. What is the sum:

$$\sum_{i=0}^{\infty} \frac{1}{2^i} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$

See you next week on Fibonacci ♥

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