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Problem 1. $P(x) = x^3 + 33x^2 + 327x + 935$ Let a, b, c be the roots of $P(x)$. Find $a^2 + b^2 + c^2$ without solving $P(x) = 0$

Problem 2. Let a polynomial $P(x)$ be defined as $P(x) = x^2 - 2x + 6$ with its (complex) roots a and b. Then what is the value of $a^{10} + b^{10}$?

Problem 3. If roots of the polynomial $x^2 + x - 1 = 0$ are a and b, what is $a^7 + b^7$?

Problem 4. If the roots of the polynomial $P(x) = x^3 - 3x^2 + 6x - 9$ are a, b and c, what is the value of $a^5 + b^5 + c^5$?

Problem 5. If the roots of $P(x) = x^3 + 3x^2 + 4x - 8$ are a, b,and c, what is the value of $a^2(1 + a^2) + b^2(1 + b^2) + c^2(1 + c^2)$?

Problem 6. Consider the cubic equation $245x^3 - 287x^2 + 99x - 9 = 0$ with roots a, b, c. If

$$\sum_{n=1}^{\infty}(a^n + b^n + c^n)$$

is of the form $\frac{m}{n}$, where m and n are coprime positive integers, what is the value of $\frac{m}{n+1}$?

Problem 7. If

$$\begin{aligned} a + b + c &= 1 \\ a^2 + b^2 + c^2 &= 2 \\ a^3 + b^3 + c^3 &= 3, \end{aligned}$$

what is the value of abc?

Problem 8. Evaluate $\left(1 + \sqrt{5}i\right)^{10} + \left(1 - \sqrt{5}i\right)^{10}$.

Problem 9. The number of ordered pairs of integers (m, n) for which $mn \geq 0$ and $m^3 + n^3 + 99mn = 33^3$ is equal to $2, 3, 33, 35, or 99$? (AHSME)

Problem 10. Given the system of equations:

$$\begin{aligned} x + y + z &= 1 \\ x^2 + y^2 + z^2 &= 2 \\ x^3 + y^3 + z^3 &= 3, \end{aligned}$$

find the smallest positive integer value of $n(> 3)$ such that $x^n + y^n + z^n$ is an integer.

Problem 11. (AIME) Suppose that the sum of the squares of two complex numbers x and y is 7 and the sum of the cubes is 10. What is the largest real value that $x + y$ can have?

Problem 12. Find x, y, and z if

$$\begin{aligned} x+y+z &= 8 \\ x^2+y^2+z^2 &= 62 \\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z} &= \frac{1}{90}, \end{aligned}$$

and $x \geq y \geq z$

Problem 13. Determine k, a positive integer if,

$$\begin{aligned} a^1+b^1+c^1 &= k \\ a^2+b^2+c^2 &= k \\ a^3+b^3+c^3 &= k \\ abc &= 5! \end{aligned}$$

Problem 14. Let $P_n = a^n + b^n$ where a and b are the roots of x^2+x+1, find the value of

$$\sum_{n=1}^{1729} (-1)^n P_n$$

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Problem 15. Find the value of $-a$ for which the roots x_1, x_2, x_3 of $x^3 - 6x^2 + ax - a = 0$ satisfy $(x_1-3)^3 + (x_2-3)^3 + (x_3-3)^3 = 0$

Problem 16. $P(x) = x^3 - 3x + 1$. $Q(x) = x^3 + Ax^2 + Bx + C$ be a polynomial with integer coefficients such that its roots are the 5th power of the roots of $P(x)$. What is the value of $A + B + C$?

Credit to Brilliant.org for subject development and most of the problems.