Strolling with Euler: A Matter of Degree

Remember that a planar graph is one which no edges cross; and recall that last week, we proved that all planar graphs satisfy the equation:

(# of vertices) – (# of edges) + (# of faces) = 2

Take a moment to explain why there are at least twice as many edges in any planar graph as there are faces, below:

On the board, I'll draw a picture of "K_5" and "$K_{3,3}$". In the space below, try to draw them without crossing any edges—as planar graphs. Try at least four times each.

For the graph "K_5," we have an argument that can be made according to the facts about the "Euler characteristic" that we learned about last week; copy what is written on the board here, so that you can refer to it later.

Now, for "$K_{3,3}$", we can see that there are no triangles in it.

Then, if we draw it without crossing edges, every face would have at least four edges. Explain below why any planar graph with no triangles should have $4f \leq 2e$, and use this to show why "$K_{3,3}$" can't be planar.

We can use similar techniques to solve the below problems:

Prove that the sum of the degrees (=# of edges coming from a vertex) of all the vertices is twice the number of edges in any graph.

At a party, guests greet each other by shaking hands. Prove that the number of guests who shake hands an *odd* number of times must be *even*.

Homework: Find out how to arrange the vertices and edges of "K_5" and "$K_{3,3}$" in 3D space so that there are no crossings.