Strolling with Euler: Trying to be Flat

We are going to call a graph “planar” if it is a diagram in which no edges cross. Draw a few planar graphs below.

Next, draw a few graphs that aren’t planar.

Often, we can turn graphs that look like they aren’t planar into planar graphs, by making an edge curved, or by moving a vertex (I will give examples on the board). Can you turn any of the graphs that you wrote above into a planar graph now?

Now, for each of the planar graphs you drew on the first page, try calculating the following number:

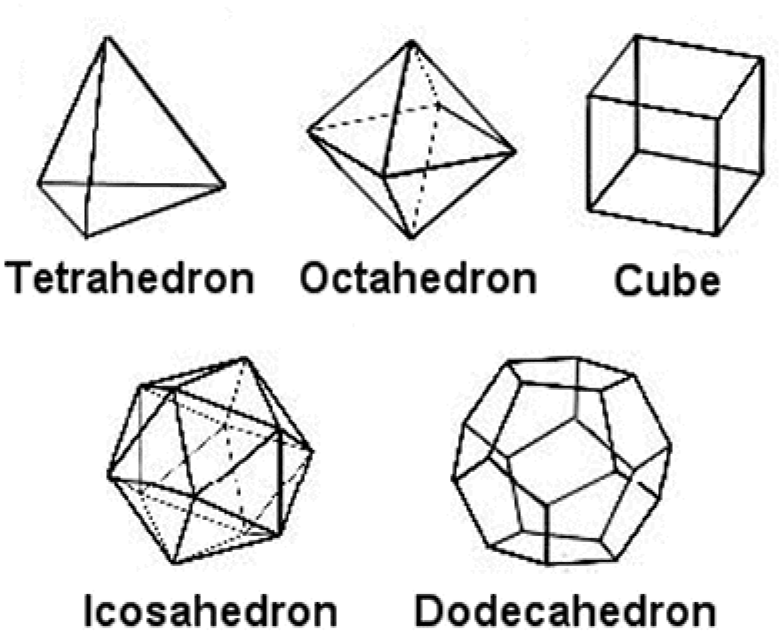
(# of vertices) – (# of edges) + (# of faces).

In this case, a face is a region surrounded by non-crossing edges. What do you notice?

The space below is reserved for taking notes from the proof of the “Euler characteristic” that I write on the board:

It is a little bit strange that we would call “graphs in which no edges cross” planar, and that we would talk about “faces” for flat objects.

We call the **tetrahedron**, **cube**, **octahedron**, **dodecahedron**, and **icosahedron** the Platonic solids[1]:



Every one of these shapes is **regular**. All the faces are the same polygon, and each face’s sides are all of the same length. A constant number of faces meets at each vertex.

Calculate the Euler characteristic for each of the polyhedra above:

Shape	Faces	Edges	Vertices	V – E + F
Tetrahedron	4			
Cube	6			
Octahedron	8			
Dodecahedron	12			
Icosahedron	20			

Actually, there are nice formulas to calculate all of the edges and vertices if you pay attention to how many shapes meet at each vertex. Can you find them?

1 Image taken from: https://3d1parsons2012.blogspot.com/2012/09/the-platonic-solids.html

Check: Draw the platonic solids as planar graphs below:

Homework: Draw some non-regular polyhedra and check the Euler characteristic. Can you find some figures that can’t be drawn as planar graphs?