Smullyan's Revenge: The Smullyaning I¹

Inspector Craig is called to France to investigate insane asylums where it is suspected that something is wrong.

- The only inhabitants of each asylum are patients and doctors.
- Each inhabitant, whether patient or doctor, is sane or insane.
- The sane ones are totally sane. They are 100% accurate in all their beliefs.
- The insane ones are totally insane. They are 100% incorrect in all their beliefs.
- Both the sane and the insane are completely honest and always tell (what they believe to be) the truth.
- 1. In the first asylum Craig visited, he spoke separately to two inhabitants whose last names are Jones and Smith.

"Tell me," Craig asked Jones, "what do you know about Mr. Smith?"

"You should call him *Doctor* Smith," replied Jones. "He is a doctor on our staff." Sometime later, Craig met Smith and asked, "What do you know about Jones? Is he a

patient or a doctor?"

"He is a patient," replied Smith.

The inspector thought over the situation for a while and then realized that something was indeed wrong with this asylum. Either one of the doctors was insane, and shouldn't be working there, or one of the patients was sane and shouldn't be there at all. How did Craig know this?

2. In the next asylum Craig visited, one of the inhabitants made a statement from which the inspector could deduce that the speaker must be a sane patient, hence did not belong there. Craig took steps to have him released.

What might the inhabitant have said?

- 3. In the next asylum, an inhabitant made a statement from which Craig could deduce that the speaker was an insane doctor. What might the inhabitant have said?
- 4. In the next asylum, Craig asked one of the inhabitants, "Are you a patient?" He replied, "Yes." Is there anything necessarily wrong with this asylum?
- 5. In the next asylum, Craig asked one of the inhabitants, "Are you a patient?" He replied, "I believe so."

Is there necessarily anything wrong with this asylum?

6. In the next asylum Craig visited, he asked an inhabitant, "Do you believe you are a patient?" The inhabitant replied, "I believe I do." Is there necessarily anything wrong with this asylum?

¹ These problems have been adapted from Smullyan's *The Lady and the Tiger*, which possesses more interesting logic puzzles like these than I could conceivably borrow.

- 7. Craig found the next asylum more interesting. He met two inhabitants, A and B, and found out that A believed that B was insane, and B believed that A was a doctor. Craig then took measures to have one of the two removed. Which one, and why?
- 8. The next asylum was a puzzler, but Craig finally managed to get down to the bottom of things. He found out that the following conditions were in place:
 - a. Given any two inhabitants, A and B, either A trusts B or he doesn't.
 - b. Some of the inhabitants are *teachers* of other inhabitants. Each inhabitant has at least one teacher.
 - c. No inhabitant A is willing to be a teacher of an inhabitant B unless A believes that B trusts himself.
 - d. For any inhabitant A there is an inhabitant B who trusts all and only those inhabitants who have at least one teacher trusted by A.
 - e. There is one inhabitant who trusts all of the patients but who does not trust any of the doctors.

Inspector Craig thought this over for a long time and was finally able to prove that either one of the patients was sane or one of the doctors was insane. Can you find the proof?

- 9. In this asylum, Craig interviewed four inhabitants: A, B, C, and D. A believed that B and C were alike as far as their sanity was concerned. B believed that A and D were alike as far as their sanity was concerned. Then Craig asked C, "Are you and D both doctors?" C replied, "No." Is there anything wrong with this asylum?
- 10. Inspector Craig found his tenth case particularly interesting, though difficult to crack. The first thing he discovered was that the asylum's inhabitants had formed various committees. Doctors and patients, he learned, could serve on the same committee and sane and insane persons might be on the same committee. Then Craig found out the following facts:
 - a. All patients formed one committee.
 - b. All doctors formed one committee.
 - c. Each inhabitant had several friends in the asylum, and among them one who was his best friend. Also, each inhabitant had several enemies in the asylum, and among them one called his worst enemy.
 - d. Given any committee, C, all inhabitants whose best friend was on C formed a committee, and all inhabitants whose worst enemy was on C also formed a committee.
 - e. Given any two committees—Committee 1 and Committee 2—there was at least one inhabitant, D, whose best friend believed that D was on Committee 1 and whose worst enemy believed that D was on Committee 2.

Putting all these facts together, Craig found an ingenious proof that either one of the doctors was insane or one of the patients was sane. How did Craig know this?