"Squares"[1]

1. Determine the value of $1 + 2 + 3 + \cdots + 2018 + 2017 + \cdots + 2 + 1$.

Three different ways of solving the above problem are:

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2. Choose any natural number and compute its square. Than add both your original number and the next higher integer to this square. What do you notice?

3. Use a picture to explain why the above trick works.

[1] This lesson was adapted from the book *Circle in a Box*, by Sam Vandervelde, pp. 131-140. Sam started the Stanford Math Circle and has led many brilliant sessions at Math Teachers' Circles around the Bay Area; the materials are often available online, and you are recommended to look them up and give his rich problem sets a try.

4. Compute the values of 35^2, 45^2, 55^2, and 65^2. What patterns do you notice in the last two digits of the answers? In the first two digits?

5. One of the numbers 212522, 213444, or 214369 is not a perfect square. Identify which one without performing any calculations.

6. Draw a picture of a 71×71 square with a 29×29 square removed. Dissect the remaining L-shaped figure into two pieces that can be reassembled into a rectangle. What does this tell you about the value of $71^2 - 29^2$?

7. Calculate the value of $20^2 + 19^2 + \cdots + 11^2 - 10^2 - 9^2 - \cdots - 1^2$ without a calculator.

8. Find the prime factorizations of 3599 and 2491.

9. Can a positive perfect square be equal to exactly twice another perfect square? Either find an example or explain why it could never occur.

10. It is possible for twice a perfect square to differ from another perfect square by exactly one. For example, $2 \cdot 5^2 = 50$, which is just above $7^2 = 49$. Find at least three other examples.

11. List your examples from the previous problem in increasing order. Then detect a pattern which will allow you to generate more examples, and check to see if your new examples work.

HW: Read a book. If you're feeling ambitious, make a reading goal for 2018, too, and stick to it.