BMC Spring 2019
Beginners I

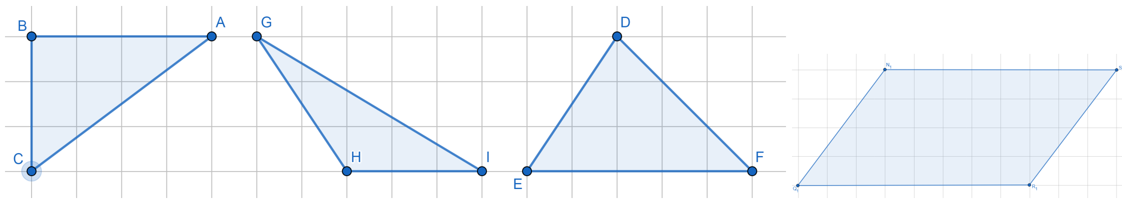
Mathe-Magical Scissors: Squaring Problems, Part II

By Harry Main-Luu

(Continued from last time)

We will start with checking our memory of the knowledge we gained last week:

1. Calculating areas of familiar shapes:

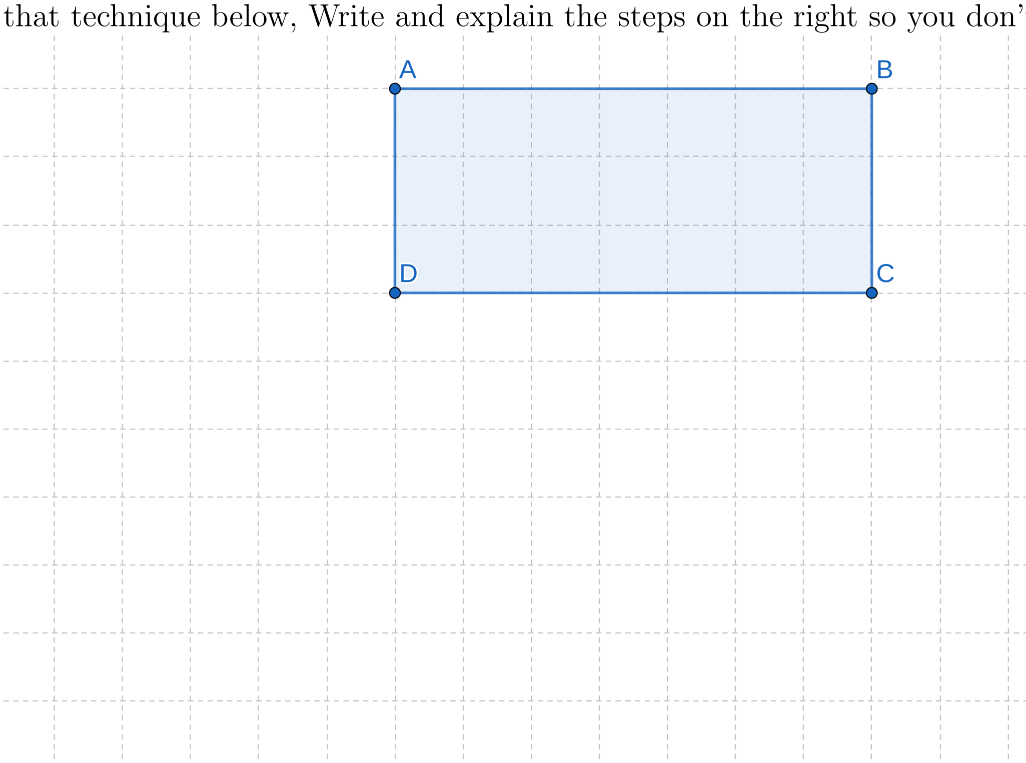


2. The sum of three interior angles of a triangle is

3. Basic properties of an isosceles triangle:

4. Harry's childhood favorite theorem: Pythagorean Theorem. What does it say?

Recall our infamous technique of decomposing an arbitrary rectangle into a square: Perform that technique below, Write and explain the steps on the right so you don't forget later.



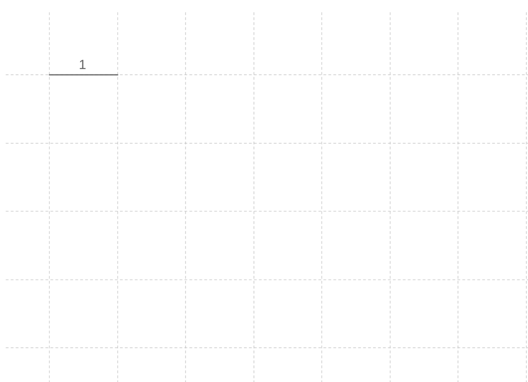
Today, we will explore why this technique works! That is, why is the highlighted (blue) length we constructed exactly $\sqrt{65}$?

Lemma 1: Any point K on a circle with diameter HC makes ΔHKC right-angled at K (where K is neither end of the diameter).

Proof

Now, the key to this decomposition is the construction of the geometric mean of two numbers. In fact, this shows that we can construct the square root of a lot of numbers! Let's explore... Fix a unit length on your graph paper, try to draw/construct the following lengths:

- $\sqrt{2}, \sqrt{3}$.
- $\sqrt{n}$, for any $n \in \mathbb{N}$.
- What about $\sqrt{\sqrt{2}}$?



Now, to return to our original question: Can we cut a certain shape and put it back into a square?
Let's try a slightly harder shape: The Triangle and the Parallelogram:

How do we cut the following triangle and parallelogram and turn them into squares?