

## Counting: Journey to Dimension $n+1$

The number of ways to select  $k$  objects out of  $n$  in no particular order is:

First, let's start by reviewing some of what you might know about coordinate geometry. The Cartesian Coordinate Plane, made by the mathematician Descartes, was a tool invented to interpret geometry through the language of algebra.

It consists of two number lines,  $x$  and  $y$ , that are perpendicular to each other. Making a grid, we can express the position of any point as a pair of numbers.

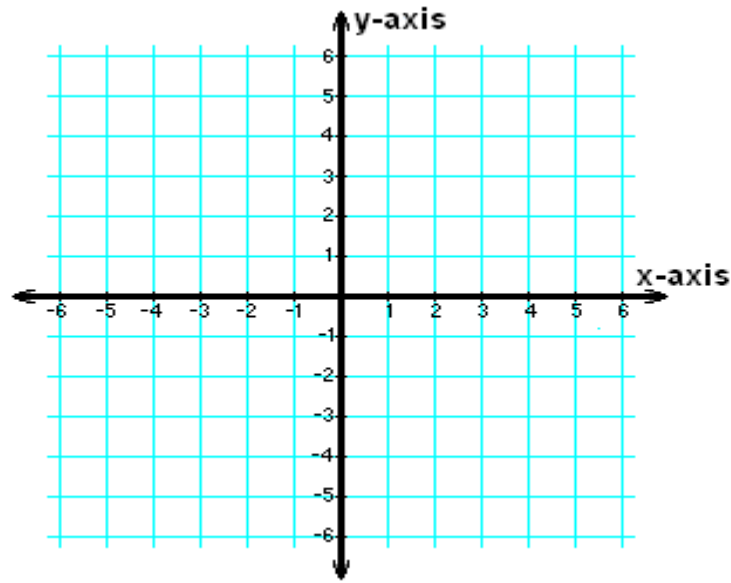
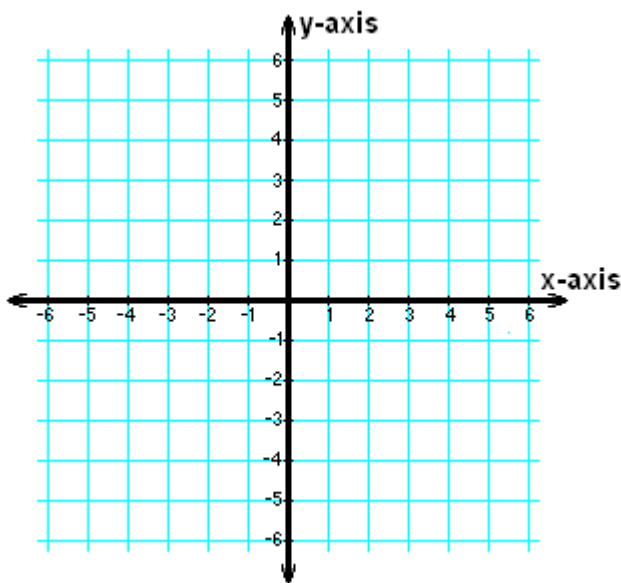
That is:

$(a, b)$  is the point that is  $a$  units along the  $x$ -axis and  $b$  units along the  $y$ -axis.

On each grid below, mark the indicated points:

**A:** (1, 0), **B:** (0, 1), **C:** (-1, 0), **D:** (0, -1)

**A:** (-4, -2), **B:** (-2, -1), **C:** (2, 1), **D:** (4, 2)



What shape does the second set of points make? Can you name some more points that are part of the same shape?

Keep everything from the last page in mind as we move on. Today, we are going to describe a **polytope series** (=way of systematically making higher-dimensional shapes from low-dimensional ones).

Can you draw a 1-dimensional shape?

Can you draw a 2-dimensional shape?

Can you draw a 3-dimensional shape?

... How about a zero-dimensional shape? Or a 4-dimensional shape?

Today we are especially interested in the **point**, **line segment**, **square**, and **cube**. Draw them below.

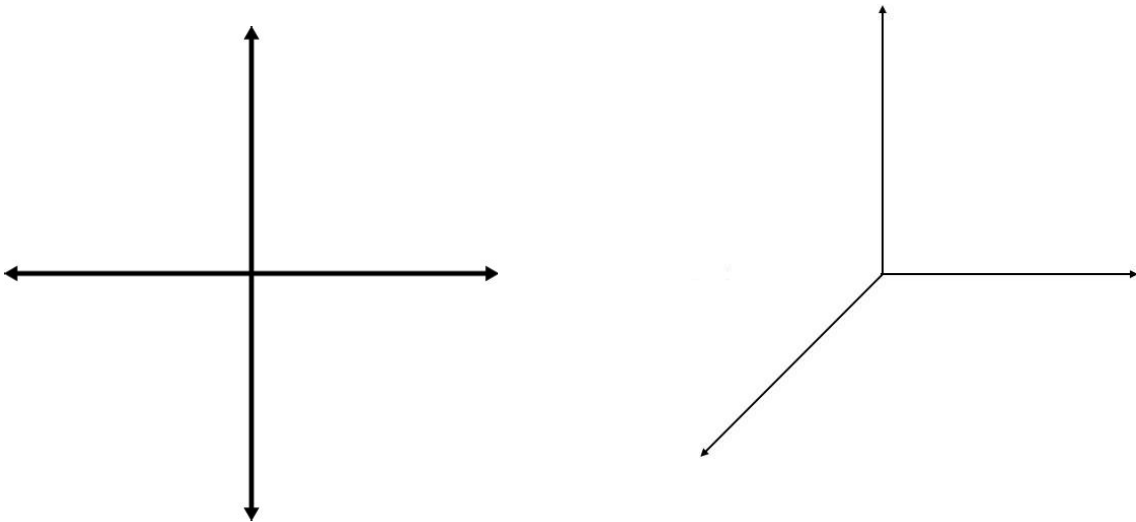
In the table below, the left column names one of the pictures you drew. For each object, count the number of each component at the top that can be found within it.

The square has already been completed for you, as an example.

	# points, or <b>vertices</b>	#segments, or <b>edges</b>	# squares, or <b>faces</b>	#cubes, or <b>3-d components</b>	
<b>point</b>					
<b>segment</b>					
<b>square</b>	4	4	1	0	
<b>cube</b>					

What should go in the row below the cube? It might be easier to count its parts if you could draw a picture...

We will answer some questions about the square and the cube (using a 3-D coordinate system!) that will help us to think about that tricky last row. Copy the pictures and coordinates drawn on the board, below.



How to calculate the number of vertices in a square:

How to calculate the number of edges in a square:

How to calculate the number of vertices in a cube:

How to calculate the number of edges in a cube:

How to calculate the number of faces in a cube:

How to calculate the number of vertices for the 5<sup>th</sup> row:

How to calculate the number of edges for the 5<sup>th</sup> row:

How to calculate the number of faces for the 5<sup>th</sup> row:

How to calculate the number of cubes for the 5<sup>th</sup> row:

HW: 1. Calculate the values for the 6<sup>th</sup> row.

2. Invent another polytope series and discover its calculation rules.