

**PERMUTATIONS, COMBINATIONS,
AND FANCY COUNTING IN THE SEA**

This space is for notes about factorials, permutations, and combinations.

SOME PERMUTATION PROBLEMS:

1. Seventeen flamboyant cuttlefish have gathered for their annual parade. How many ways can they line up single file?
2. Seven stringrays like to swim together in a V formation. How many ways can they arrange themselves?
3. Say 2 of the 7 stingrays above are spotted, and 5 are not. If one of the spotted stingrays must be at the tip of the V, how many ways can they arrange themselves?
4. Gobies and (blind) pistol shrimp have a symbiotic relationship (the shrimp builds their home and the goby provides protection). If there are 8 gobies and 8 pistol shrimp, how many ways can they pair off (one shrimp and one goby in each pair)?
5. Thirteen great white sharks are having a race. How many possibilities are there for the winners' podium – 1st, 2nd, and 3rd place?
6. The trumpetfish marching band has 16 members, and they start each performance in a 4×4 grid. How many ways can they arrange themselves if there are no restrictions on which trumpetfish can be in which positions?
7. Let's modify the previous problem a bit. Say 8 of the trumpetfish are yellow, and 8 are red. They will still arrange themselves in a 4×4 grid, but now they insist on a checkerboard pattern. How many options do they have?
8. Let's go even crazier. Suppose the band will arrange just 9 of the 16 trumpetfish into a 3×3 checkerboard pattern for a special song. How many ways can they do this?
9. How many ways can we rearrange the letters in the word FISH? How about HUMPBACK or SEA LION?
10. What about OTTER or TURTLE or CUTTLEFISH or MANTIS SHRIMP or BASKING SHARKS or MOLA MOLA? (These are trickier.)

SOME COMBINATION PROBLEMS:

11. You can only shake hands with yourself one way, since you only have 2 hands. However, most squids have ten tentacles. How many different tentacle shakes can a squid do with itself (i.e. one tentacle shaking with one other tentacle)?
12. A scuba diving club has 10 members. At their next meeting, they will select a council of 3 members who will be in charge of organizing events. How many possibilities are there for the council?
13. A pod of 15 orcas is trying to steal fish from two fishing boats. The fishermen try to outsmart the orcas by splitting up, but then the orcas decide to split up too! How many ways can the orcas divide themselves into two groups, if one group has 6 members and one group has 9 members?
14. A decorator crab has 15 spots on its shell that he would like to fill with pink anemones, pieces of kelp, and tiny shells. How many different patterns can he make using 8 pink anemones, 4 pieces of kelp, and 3 tiny shells? (Assume all the anemones are identical, all the pieces of kelp are identical, and all the shells are identical.)
15. A treasure chest has 10 different precious gems in it, and their values are \$1, \$2, \$3, \$4, \$5, \$6, \$7, \$8, \$9, \$10. An octopus reaches in and takes one gem with each of his eight arms. How many possibilities are there for which collection of gems the octopus picks up? Is this the same as asking how many possibilities are there for the total value of his collection?

ALL SORTS OF COUNTING PROBLEMS:

16. The penguin high council has gathered for their monthly meeting. The 5 council members will sit in a circle, and they only care about their relative order around the circle. That is, it matters who is to the right and left of each penguin, but if they were to all rotate around the circle simultaneously, they would consider that the same arrangement. How many different ways are there to seat the whole council?
17. A musical octopus has a tiny piano with only 12 keys. How many different “songs” consisting of 10 notes can the octopus compose?

18. A paranoid octopus has decided to put a combination lock on his den, so nobody can steal his stuff. If the lock has a passcode consisting of 2 letters followed by 3 digits, how many passcodes can the octopus choose from?
19. The Seahorse Express is how ocean animals send mail. The San Francisco Bay branch employs 16 seahorses. How many ways can they assign jobs to the seahorses, if they need 11 messengers, 2 mail sorters, and 3 customer service agents?
20. There are twelve holes in a row that moray eels can hide in. If there are 5 green moray eels, and 7 spotted moray eels, how many ways can they choose hiding holes so that the green eels are all one side, and the spotted eels are on the other.
21. Let's modify the previous problem a bit – what if the green eels need to stay lined up together, but the spotted eels may or may not be split up?
22. Three sea otter buddies (Ada, Ben, and Coo) go hunting for sea urchins. Together, they found a total of 6 sea urchins. Each otter found at least one urchin. How many different ways could their catch have been distributed? (For example, one possibility is Ada found 3, Ben found 1, and Coo found 2.)
23. What if we didn't know each otter above found at least one urchin? (So Ada 0, Ben 5, Coo 1 would be a possibility.) Then how many ways could their catch be distributed?
24. Some frogfish can walk on 2 legs on the ocean floor. Francy is a fancy frogfish who likes to wear socks and shoes. She has a left sock and a left shoe, as well as a right sock and a right shoe. How many ways can she put on her socks and shoes? (Note: as with human socks and shoes, Francy's socks must go under the shoes, so "right shoe, right sock, left sock, left shoe" is an ordering she CANNOT pick.)