GCD, LCM, AND REMAINDERS

First a bit of warmup:

- What is the definition of the GCD (greatest common divisor) of two positive integers?
- How do you find it?
- What is the definition of the LCM (least common multiple) of two positive integers?
- How do you find it?
- Let's try some examples.
- Here's a cool fact: If x and y are two positive integers, then $gcd(x, y) \cdot lcm(x, y) = x \cdot y$. Check that it works on our examples above.

- Next we will discuss a very efficient way to find GCDs. This is called the *Euclidean Algorithm*. It is easiest to explain by examples, so we will do a couple together.

PROBLEMS USING GCD AND LCM: Most of these problems are from Alcumus on www.artofproblemsolving.com, which you can try yourself. These get extremely tough near the end.

1. Compute $\gcd(348, 99)$ by factoring and by the Euclidean Algorithm.
2. Compute $\gcd(6^2 10^2, 15^4)$.
3. Compute $lcm(6^2 10^2, 15^4)$.
4. Let $m = \underbrace{22222222}_{\text{8 digits}}$ and $n = \underbrace{444444444}_{\text{9 digits}}$.

   What is $\gcd(m, n)$?
5. Find the greatest common divisor of 957 and 1537.
6. Find the greatest common divisor of 2863 and 1344.
7. Find the greatest common divisor of 3339, 2961, and 1491.
8. The least common multiple of $1! + 2!$, $2! + 3!$, $3! + 4!$, $4! + 5!$, $5! + 6!$, $6! + 7!$, $7! + 8!$, and $8! + 9!$ can be expressed in the form $a \cdot b!$, where a and b are integers and b is as large as possible. What is $a + b$?
9. Let $a_n = \frac{10^n - 1}{9}$. Define d_n to be the greatest common divisor of a_n and a_{n+1}. What is the maximum possible value that d_n can take on?
10. The greatest common divisor of positive integers m and n is 6. The least common multiple of m and n is 126. What is the least possible value of $m + n$?