DIVISIBILITY AND OTHER FUN WITH INTEGERS

First, let's talk about some nice divisibility rules and why they work. Sometimes the rules are quite simple. Sometimes they are a little difficult, but still easier than doing a really big long division.

Let's start with a few easy ones:

- How to check if a number is divisible by 2:
- Why does it work?

- How to check if a number is divisible by 5:
- Why does it work?

- How to check if a number is divisible by 10:
- Why does it work?

Now a bit fancier. The tricks for these use adding.

- How to check if a number is divisible by 3:
- Why does it work?

- How to check if a number is divisible by 9:
- Why does it work?

- How to check if a number is divisible by 11:
- Why does it work?

How do things go with bigger multiples of 2?

- How to check if a number is divisible by 4:
- Why does it work?

- How to check if a number is divisible by 8:
- Why does it work?

- How to check if a number is divisible by 16:
- Why does it work?

- How to check if a number is divisible by 2^N:
- Why does it work?

How can we combine our previous answers and explanations?

- How to check if a number is divisible by 6:
- Why does it work?

- How to check if a number is divisible by 15:
- Why does it work?

- How to check if a number is divisible by 30:
- Why does it work?

- How to check if a number is divisible by 18:
- Why does it work?

Do you think there are divisibility rules for other numbers? What else can we try?

Let's test our rules on some large integers. Room for scratch work below:

Now for some challenge problems related to divisibility:

1. Fill in the smallest digit to make the number divisible:
   (i) by 5 : 7164__, 32197__
   (ii) by 3 : 1__43, 47__05, __316
   (iii) by 6 : __428, 9__52, 721__
   (iv) by 4 : 2462__, 91__ __, 670__
   (v) by 8 : 1232__, 59__16, 4642__
2. LOTS of four digit multiples of 11 have a digit sum of 12. How many can you find?
3. What is the largest six-digit number that is NOT divisible by 2, 3, 5, or 11?
4. Find the largest four-digit palindrome that is divisible by 3.
5. Find the smallest four-digit palindrome that is divisible by 3.
6. Find the largest five-digit palindrome that is divisible by 6.
7. What is the smallest positive multiple of 450 whose digits are all zeroes and ones?
8. What is the least positive integer n such that 80325 divides $n!$?
9. Alex is at the candy store buying jellybeans. He wants to buy at least 100 jellybeans. He wants to buy the least amount such that he would have exactly 11 leftover after dividing the jellybeans evenly among 13 people. How many jellybeans should Alex buy?
10. For some positive integer k, when 60 is divided by k^2, the remainder is 6. What is the remainder when 100 is divided by k?