FRIEZE PATTERNS

Berkeley Math Circle
September 19, 2018

Frieze 1

$$
\begin{array}{ccccccccccccccc}
 & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & \\
\cdots & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & \cdots \\
 & 2 & & 2 & & 1 & & 3 & & 1 & & 2 & & 1 & \\
\cdots & & 3 & & 1 & & 2 & & 2 & & 1 & & 3 & & \cdots \\
 & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & \\
\cdots & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & \cdots
\end{array}
$$

Frieze 2

$$
\begin{array}{ccccccccccccccc}
 & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & \\
\cdots & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & \cdots \\
 & 1 & & 3 & & 1 & & 3 & & 1 & & 3 & & 1 & \\
\cdots & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & \cdots \\
 & 3 & & 1 & & 3 & & 1 & & 3 & & 1 & & 3 & \\
\cdots & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & \cdots \\
 & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 &
\end{array}
$$

Frieze 3

$$
\begin{array}{ccccccccccccccccc}
 & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & \\
\cdots & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & \cdots \\
 & 4 & & 2 & & 1 & & 3 & & 2 & & 2 & & 1 & & 3 & \\
\cdots & & 7 & & 1 & & 2 & & 5 & & 3 & & 1 & & 2 & & \cdots \\
 & 5 & & 3 & & 1 & & 3 & & 7 & & 1 & & 2 & & 5 & \\
\cdots & & 2 & & 2 & & 1 & & 4 & & 2 & & 1 & & 3 & & \cdots \\
 & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & \\
\cdots & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & \cdots
\end{array}
$$

Frieze 4

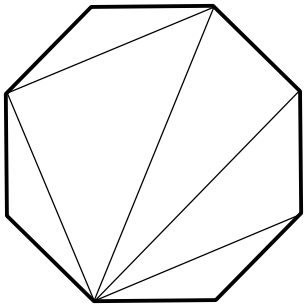
$$
\begin{array}{ccccccccccccccccccccc}
 & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & \\
\cdots & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & \cdots \\
 & 1 & & 4 & & 2 & & 1 & & 3 & & 5 & & 1 & & 2 & & 2 & & 3 & & 1 & \\
\cdots & & 3 & & 7 & & 1 & & 2 & & 14 & & 4 & & 1 & & 3 & & 5 & & 2 & & \cdots \\
 & 5 & & 5 & & 3 & & 1 & & 9 & & 11 & & 3 & & 1 & & 7 & & 3 & & 1 & \\
\cdots & & 8 & & 2 & & 2 & & 4 & & 7 & & 8 & & 2 & & 2 & & 4 & & 7 & & \cdots \\
 & 11 & & 3 & & 1 & & 7 & & 3 & & 5 & & 5 & & 3 & & 1 & & 9 & & 11 & \\
\cdots & & 4 & & 1 & & 3 & & 5 & & 2 & & 3 & & 7 & & 1 & & 2 & & 14 & & \cdots \\
 & 5 & & 1 & & 2 & & 2 & & 3 & & 1 & & 4 & & 2 & & 1 & & 3 & & 5 & \\
\cdots & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & \cdots \\
 & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 &
\end{array}
$$

Problem 1. Examine friezes on the previous page, what patterns do you notice?

Problem 2. Can you build/construct some friezes of your own? What problems do you encounter? What is the most difficult condition to satisfy?

Problem 3. What is the least amount of data needed to compute a frieze?

Problem 4. Construct the frieze coming from the polygon below.

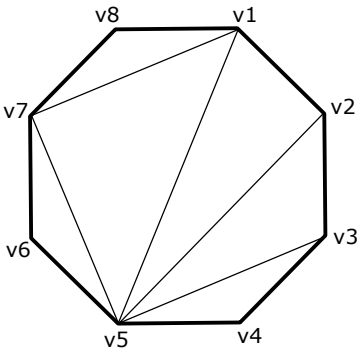


Problem 5. How are the size of the polygon and the width of the frieze related?

Problem 6. Can we go backwards? For each of the friezes on the first page come up with a triangulation.

Problem 7. What is the number of entires in a frieze up to symmetry? What does this number correspond to in the polygon?

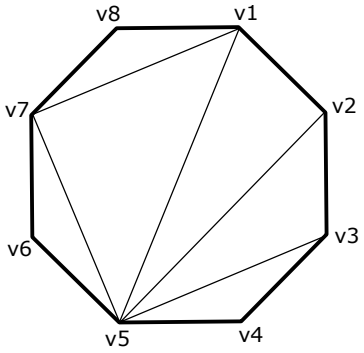
Problem 8. Draw the diagonals grid for the polygon below. Compare it with the frieze you constructed above. What do the 1's in the frieze correspond to in the polygon?

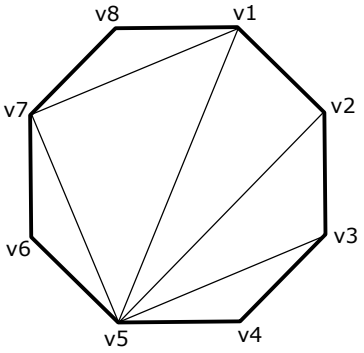


Problem 9. How does the polygon model explain the symmetry of the frieze?

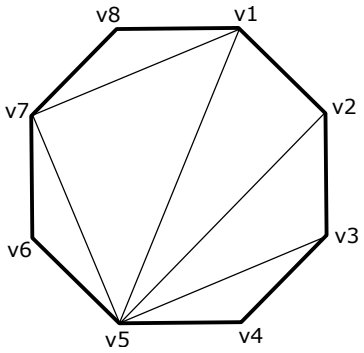
Problem 10. Using "lengths" method, compute the entegers corresponding to the entries $(4, 7)$ and $(4, 8)$. Do they agree with the numbers in the frieze you computed?

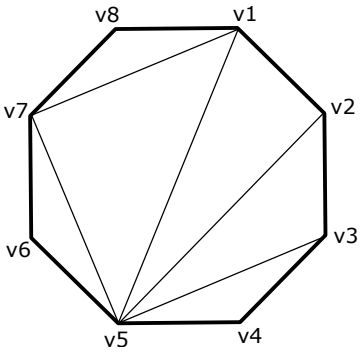
Problem 11. Using "counting game" method, compute the integers corresponding to the entries $(2, 6)$ and $(4, 8)$. Do they agree with the numbers in the frieze?





Problem 12. Using "counting triangles" method, compute the integers corresponding to the entries $(4, 6)$ and $(3, 7)$. Do they agree with the numbers in the frieze?





Problem 13. Using "counting substrings" method, compute the integers corresponding to the entries $(3, 6)$ and $(4, 8)$. Do they agree with the numbers in the frieze?

Problem 14. Using "snake graph" method, compute the integers corresponding to the entries $(3, 6)$ and $(4, 8)$. Do they agree with the numbers in the frieze?

Problem 15. What are your favorite methods? Why? Discuss advantages and disadvantages of each.

Problem 16. What are the numbers in the frieze coming from zig-zag triangulations? Which method can you use to justify this?