BMC: Taxicab Metric

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I. Preamble

What is Taxicab Metric?

The usual notion of distance between two points in the plane is quite simple: the length of the segment connecting them. In real world, that would mean the shortest way to go from point A to point B.

In your graph paper, plot points $A(3,0)$ and $B(0,4)$ and draw this distance. If each unit in your graph paper corresponds to 10 ft in real life, how far would I need to walk from A to B? (Perhaps we need a famous theorem whose name starts with a P)

But really, in real life, we have buildings and obstacles and traintracks to avoid. It isn't always possible to walk straight from point A to B. Think of the grid on your graph paper as a perfect city where everyline is a street and every intersection point is a corner of a street. There are buildings and houses everywhere. Only a bird can fly from point A to point B without obstructions! We will need a different type of distance to measure and find what it the shortest way from A to B. Any guess?

II. Fun Ideas and Applications

1. On graph paper, mark the following pairs of points and find their taxi distance:
   a) P(5,4) and Q(1,2)
   b) A(-3,2) and B(-1,-1)
   c) A as above and C(-3,5)
2. When are two points have the same Euclidean distance and Taxicab distance?
3. Graph all points with taxi distance 3 from A. What should we call the resulting shape?
4. Think some more
5. What would be a reasonable value for π in Taxicab Geometry?
6. Find some points P that have the same distance to A and B below:
   a) A(0,0); B(4,2)
   b) A(-3,2); B(-3,5)
   c) A(0,0); B(4,4) – Be careful!
   d) What should we call the collection of all the points equidistant to A and B?
7. In the Perfect City (with all streets laid out in perfect square grid), the Post Office is at the origin. Alexander goes to Atlantic school at A(-3,-1) and Bobby works bakery B at (3,3). They want to find an apartment to live together and will walk everywhere. They agree that their apartment should be somewhere that the total distance of their walk altogether should be minimal.

a) Where should they look for the apartment?

b) Alexander just signed up to a 7am class. Not wanting to be late, they decide Alex shoud not have to walk any further than Bobby does. Where can they look for their apartment now?

c) Bobby got promoted and now has to open the bakery early in the morning as well. They decide they should walk the same distance. Now where can they look?

d) Yay! They found an apartment in that ideal location! But it's too expensive...

e) While they are figuring out their finances, the 9-1-1 dispatcher for Perfect City receives a report of an accident at (-1,4). There are two police cars in the area, car C at (2,1) and car D at (-1,-1). Which car should the dispatcher send to the scene of the accident?

8. There are three middle schools in Perfect City. Franklin at (2,1), Lincoln at (-4,3) and Washington at (-1,-6). Can you help Mayor Hope to zone the city into school districts so that every student attends the school closest to their home?

9. An icecream truck wants to travel on the best route when all three schools let out at 3pm, that is a route that is equidistant to all three schools. Where can the route be?

10. The ice cream truck owner, Euclid, later learns that Franklin lets out at 2pm, Lincoln lets out at 3pm, and Washington at 4pm every weekday. He wants to find a route that would stay closest to Franklin between 2-3pm, then Lincoln 3-4pm. and so on. Naturally, he wants to be at the closest to both F and L (preferably closer to Lincoln) at 3pm. Can you find him such a route?

11. A cellphone provider wants to build a cell tower so that all three school get roughly equal signal strength. If we know that signal strength depends on the distance the waves have to travel, where should the cell tower be built? – Be careful!

III. Distance between a point a line

12. Notion(s) of distance between a point and a line in Euclidean geometry?

13. Can we copy those ideas? Draw line l through $A(-2,-1)$ and $B(2,3)$. Find a few more points on line l.
    a) Find the distance between point $H(-3,2)$ and all the points you already have on l.
    b) Which one is smallest?
    c) Can you find the absolutely smallest distance? What should we call this number?
    d) Draw a circle center at H, radius 1.
    e) In crease your radius by 1 unit at a time until you reach the line l. What do you notice?

14. Fix a point $P(1,1)$ on the grid. Draw three lines (all passing through P) l_1 through $(2,2)$, l_2 through $(2,3)$, l_3 through $(3,2)$.
    a) For point $A(4,5)$, find distance between A and all three lines.
    b) What do you notice about the way we should find "walk" towards the line?
    c) Do these conditions depend on the point P on the line?

15. Remember Alexander and Bobby in Perfect City? Bobby found a new job as a conductor of the city newbuilt subway, along the line that passes through $(6,0)$ and $(0,2)$. Part of Bobby's awesome job perks is that he can hop on the subway anywhere nearest to where he lives. So they start on a new apartment hunting!
    a) Where should their apartment be so that the TOTAL distance Alex has to work and Bobby has to walk is minimum?
    b) What about just same distance of walking for both?
    c) How about if they individually only want to walk at most 6 blocks?
    d) What about together they walk at most 6 blocks?