Counting: Partitions

We may know how to solve problems like this one already:

For how many triplets of x, y, and z can we write $x + y + z = 7$?

This week, we will ask some other questions along similar lines. Try the following:

For how many x, y, and z can we write $x + y + z = 7$, with $x \geq y \geq z$?

As you can see, your answer to this new problem will be different than your answer to the first one. We will call these ordered collections of numbers adding to a number of interest **partitions**.[1]

If we have, for example, $5 + 4 + 3 = 12$, we will call (5, 4, 3) a partition of 12, and we can draw a picture of it by drawing 5 squares above 4 squares above 3 squares, all lined up as in the picture on the board:

Draw pictures using blocks of all of the partitions of 7:

Draw the partitions of (4, 4, 3, 1, 1) and of (5, 3, 3, 2). What is the relationship between the drawings? What number does (4, 4, 3, 1, 1) partition, and what number does (5, 3, 3, 2) partition?

1 The following material has been adapted from Kenneth Bogart's *Combinatorics Through Guided Discovery* problem #165, available at math.dartmouth.edu/news-resources/electronic/kpbogart/ComboNoteswHints11-06-04.pdf.

Draw partitions of (3, 3, 1) and of (2, 1, 1). Can you see a relationship between the drawings?

We will call a partition **self-conjugate** if, when it is flipped over as above, we will get the same diagram as we started with.

Draw some self-conjugate partitions:

Now, we will find a neat relationship between **self-conjugate partitions of a number k** and **the number of partitions of k into distinct odd parts**.

Your work will be **much easier** if you can find a way to organize it before we continue. Together, we will:

Draw the partitions of 1:

Draw the partitions of 2:

Draw the partitions of 3:

Draw the partitions of 4:

Draw the partitions of 5 (there are 7):

Draw the partitions of 6 (there are 11):

Draw the partitions of 7 (there are 13):

Draw the partitions of 8 (there are 22):

Now, go back and circle **all of the partitions you drew that are self-conjugate**.

Put a **star** next to all the partitions you drew that have **only odd parts**, and so that **no two of those parts are the same size**.

What do you notice?

Why is your observation true?

Homework: Draw a partition of 7 into 4 parts in a 4x10 box. What number is partitioned by the **complement** of your partition of 7? Use this experiment to show that any number **k** has as partitions into 4 parts, as does **3k**.