Strolling with Euler: Climbing Stairs

1.
    a. You are standing in front of a set of stairs. What is the first step you need to do in climbing the stairs?
    b. Suppose you are on stair n. How can you climb onto stair n+1?

This kind of thinking is called “inductive reasoning.” If we know a statement is true for a “base” case—like knowing how to climb onto the first step of a staircase—then we assume we know it is true for a case “n” (we know how to climb to stair n). After that, we prove that we can climb to the next stair, number n+1. “Induction” means that, if we satisfy these two steps, then we can assume the statement is true for any n (we can climb a set of stairs of any non-infinite height).

2. Prove that 1 + 2 + 3 + … + n = n(n+1)/2.
    a. Check that it is true for n = 1.
    b. Suppose that 1 + 2 + 3 + … + n = n(n+1)/2. Then show that 1 + 2 + 3 + … + n + (n+1) = (n+1)([n+1] + 1)/2.
3. Prove that if we have n vertices and connect all of them with an edge, we will have n(n+1)/2 edges there.
4. Suppose that we have n vertices, all of which are connected into a circle. Prove inductively that there are n edges to connect the n vertices.
5. Suppose there are 2n children, and separate them evenly into a blue team and a red team. At the end of the game, no matter who won and who lost, every child from the blue team must shake hands with every child from the red team. How many handshakes were there?

With induction, it is also not-so-hard to solve problems like those below:

1. How many ways are there to arrange a potato, a tomato, and a purple flower in a row?
2. What if, above, we add a hammer to the number of objects—how many ways are there to arrange them? Find some way to separate the arrangements into different groups.
3. Now, prove using induction: how many different ways are there to arrange n different unique objects?
4. More difficult: suppose there is a roller coaster with n cars, and n women and n men would like to sit there. In each car one person should be on the left, one on the right. How many possible seating arrangements are there?
5. Also difficult: suppose that n people would like to sit around a round table, and we say seating arrangements are the same if the person to everyone’s *right* remains the same. How many different seating arrangements are there?
6. After the soccer game, the blue team and red team would like to eat pizza together. Their coach says that they must sit blue, red, blue (etc.) around a round table, as in the 5^{th} problem above, in order to make them more friendly after the game. How many different seating arrangements are there?

Finally: Using induction, prove that the sum of the degrees of vertices of a graph is twice the total number of edges present in the graph.