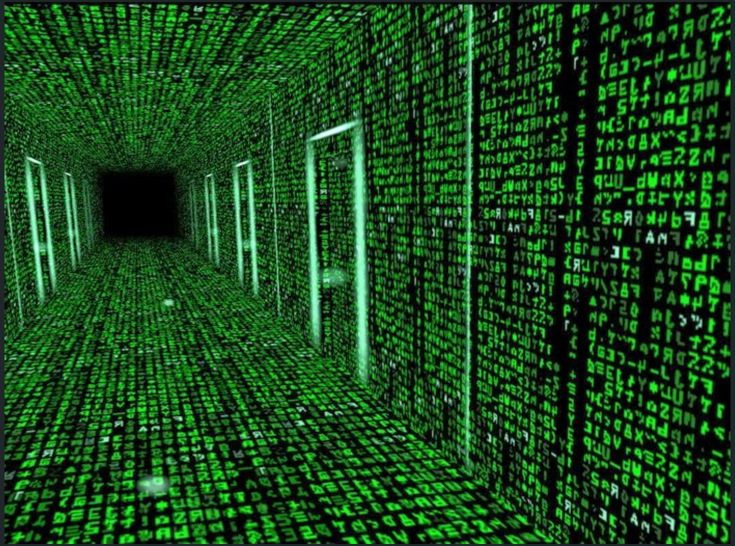
Matrices



Ellen Kulinsky

Amusement Parks

At an amusement park, each adult ticket costs $10 and each children's ticket costs $5. At the end of one day, the amusement park as sold $200 worth of tickets. You also know that in total 30 tickets were sold. How many adult tickets and how many children tickets were sold?

Money equation:

$10a + 5c = 200$

Num. of tickets equation:

$a + c = 30$

$a = 30 - c$

Substitution!

$10(30 - c) + 5c = 200$

$300 - 10c + 5c = 200$

$100 = 5c$

$c = 20$

$a = 10$

But that wasn't bad... Time to level up.

Get rid of y!

$$\begin{cases} 18x + 6y = 96 \\ 5x + 3y = 36 \end{cases} \xrightarrow{\times 2} \begin{cases} 18x + 6y = 96 \\ 10x + 6y = 72 \end{cases}$$

Subtract:

$8x = 24 \longrightarrow x = 3$

Plug back in $\longrightarrow$

$5\times3 + 3y = 36$

$15 + 3y = 36$

$3y = 21$

$y = 7$

What about now?

$$\begin{cases} 9x + 6y + z = 96 \\ 3x + \quad + 12z = 22 \\ x + 5y + 2z = 17 \end{cases}$$

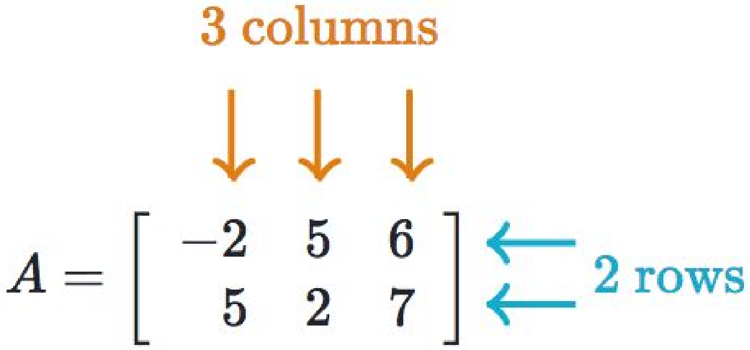
Substitution or Elimination…

is there another way?

YES! It’s called a matrix.

Matrix:

- a rectangular arrangement of numbers into rows and columns
- very useful way to represent information and work with data
- often used in computers



Dimensions: m by n matrix (rows by columns) ($m \times n$)

How does this have to do with systems of equations?

$$\begin{cases} 10a + 5c = 200 \\ a + c = 30 \end{cases} \rightarrow \left(\begin{array}{cc|c} 10 & 5 & 200 \\ 1 & 1 & 30 \end{array}\right)$$

$$\begin{cases} 18x + 6y = 96 \\ 5x + 3y = 36 \end{cases} \rightarrow \left(\begin{array}{cc|c} 18 & 6 & 96 \\ 5 & 3 & 36 \end{array}\right)$$

<u>Question</u>: Dimensions?

<u>Answer</u>: 2 by 3

<u>Question</u>: How does this relate to original equations?

<u>Answer</u>:
# of rows = # of equations
# of columns = 1 + # of variables

<u>Question</u>: Why plus 1?

<u>Answer</u>: One column represents no variables

Your turn!

$$
\begin{aligned}
3x - 2y &= 4 \\
x + 5z &= -3 \\
-4x - y + 3z &= 0
\end{aligned}
$$

$$
\begin{aligned}
3x + (-2)y + 0z &= 4 \\
1x + 0y + 5z &= -3 \\
-4x + (-1)y + 3z &= 0
\end{aligned}
$$

$$
\begin{bmatrix}
3 & -2 & 0 & 4 \\
1 & 0 & 5 & -3 \\
-4 & -1 & 3 & 0
\end{bmatrix}
\begin{matrix}
\leftarrow \text{Eq. 1} \\
\leftarrow \text{Eq. 2} \\
\leftarrow \text{Eq. 3}
\end{matrix}
$$

$$
\begin{matrix}
\uparrow & \uparrow & \uparrow & \uparrow \\
x & y & \text{z} & \text{constants}
\end{matrix}
$$

Your turn!

$$\left.\begin{aligned} 51x + 25y &= 101 \\ x + 34y &= 69 \\ 4x + 18y &= 40 \end{aligned}\right\} \quad \left(\begin{array}{cc|c} 51 & 25 & 101 \\ 1 & 34 & 69 \\ 4 & 18 & 40 \end{array}\right)$$