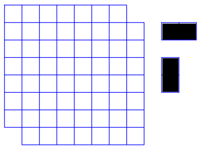
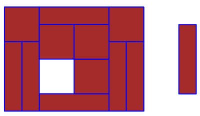
Puzzles

February 20, 2017

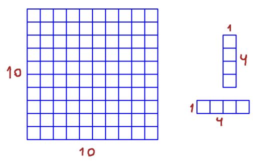
1. Consider an 8×8 checkers board, without the top left and bottom right squares. Is it possible to tile it using just 1×2, and 2×1 pieces?



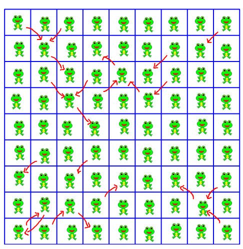
2. A tiled 6×8 board is shown in the figure below. One of the 2×2 pieces has been stolen. Luckily, a 4×1 piece was found instead. Can you retile the board with the 4×1 piece?



3. Is it possible to tile a 10×10 tile using 1×4 and 4×1 pieces?



4. 81 frogs are located on a 9×9 board such that no two frogs share a square. On midnight all the frogs **must** jump, diagonally, to a neighboring square. Can you direct the frogs how to jump, so that no two frogs will share a square after jumping?



5. Lisa and Bart play the following game. A list of 12 numbers is written on the whiteboard; each player, in its turn, picks a numbers either ends of the list and erases it. Let L be the sum of the numbers picked by Lisa and B be the sum of numbers picked by Bart. The winner is the player who has a larger sum.

6. Find a common theme to puzzles 1 — 5.
7. Lisa and Bart play the following game. Lisa picks two integers n, m, writes them down, and hides them in two envelopes. Then Bart is allowed to pick one envelope and reveal the number in it. He then has to guess which of the two numbers is bigger (without seeing the other number). Clearly, if he bets randomly, then he'll be correct with probability $1/2$. Can you help him achieve better chances?
8. n red points and n blue points are placed on the plane. Can you match the n red points to the n blue points in a way that the intervals connecting matched points do not intersect?
9. Given an $m \times n$ array of numbers, can you transform it to an array such that each row and each column has a nonnegative sum, under the constraint that the only allowed actions are: (i) negating the signs of a whole row, and (ii) negating the sign of a whole column?
10. Find a common theme to puzzles 8,9.
11. Show that among any 5 points in a 2×2 square there must be two points whose distance is at most $\sqrt{2}$.
12. Let n be a natural number that is not divisible by 2 nor 5. Show that there exists a natural number k such that $k \cdot n = 11\ldots1$
13. Let $x_1, x_2, \ldots, x_{10}$ be natural numbers between 1 and 100. Show that there are two disjoint subsets with the same sum.
14. Show that every sequence of a 100 integers contains a non empty subsequence whose sum is divisible by 100.
15. Find a common theme to puzzles 11 — 14.
16. Alice and Bob play the following game on a graph G. Alice picks a starting vertex v. Then, at alternating turns, each player picks a neighbor of the previously chosen vertex that has not been chosen so far. The loser is the player who has nowhere to go at their turn. Show that if the number of vertices is odd then Alice has a winning strategy.