Counting: Keeping things in order [Review]

You may remember, from the fall, the answer to the question:

How many ways are there to arrange three different pieces of fruit: an apple, an orange, a banana?

Draw the different arrangements below:

There are __________ arrangements.

Your older sister tells you that a tomato is also a kind of fruit, and that you should arrange it *also*.

How many ways are there to arrange the apple, orange, and banana from the first part, along with the tomato?

Draw the different arrangements below:

There are _________ arrangements.

There is a relationship between your answers to the first and second parts. Using that relationship, we can find a rule that would allow us to arrange any number **n** of objects. Write it below:

The special notation for rearranging **n** different objects is: ___________

Try these problems:

1. A pets magazine wants to publish articles on 13 different pets this season. If the company has already decided which pets will be featured, how many different ways are there to order the pets, one per week?
    a. What if they have already decided to feature kittens first and hedgehogs last?
    b. What if, instead, they have decided to feature canaries immediately before ducks?

2. How many 9-digit numbers can be obtained by using each of the digits 1, 2, … 9 exactly once? How many of these are bigger than 600,000,000?

Now, a different problem that we can *also* solve by rearranging.

SITUATION A

Suppose that you want to have a movie night with some of your friends. There is space for *only* 4 friends to sit on the couch [you will sit in a comfy armchair], and—you are friends with the 20 other members of your class. Clearly 16 friends will need to be excluded this time.

How many different combinations of friends can you invite to your house to watch the movie?

Write down two different ideas of how you can solve this problem:

1.

2.

Write down the rearrangement I put on the board:

SITUATION B

How would the result of this problem change if the friends were also picky about who sits on the left, right, and middle of the couch?

When we have **n** objects and we would like a group **k** of them in no particular order, we will call this a **combination**.

The formula for the number of groupings in this situation is:

When we have **n** objects and we would like an ordered group **k** of them, we will call this an **ordered selection**.

The formula for the number of groupings in this situation is:

One last situation. Suppose that you would now like to rearrange two oranges, two apples, and a banana. You can't tell the difference between the oranges, nor between the apple s.

Try drawing the situation below.

The number of rearrangements is:

In this case, the oranges are **indistinct**, as are the apples. We will divide the total number of rearrangements by the number of rearrangements of the oranges (2) and the number of rearrangements of the apples (2), because the situations in which the apples are exchanged with each other don't matter.

Try this problem: How many ways are there to arrange on the shelf 4 copies of an algebra book, 6 copies of a geometry book, and all 7 different volumes of Harry Potter?

Homework:

The number of ways to rearrange the letters in COUNTING is:

The number of ways to rearrange the letters in REARRANGE is:

The number of ways to rearrange the letters in COMBINATORICS is:

Suppose you need to seat **n** people around a table. Show that it can be done in $(n-1)!$ ways.

Show that $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$. Try to do it in two different ways.