Counting: Finding Subgroups

The number of ways to arrange **n** different objects is:

Write in your own words an explanation of how many ways there are to arrange **n** objects if some of them are the same:

Now, a different problem that we can *also* solve by rearranging.

SITUATION A

Suppose that you want to have a movie night with some of your friends. There is space for *only* 3 friends to sit on the couch [you will sit in a comfy armchair], and—you have 9 friends, some of whom will need to be excluded this time.

How many different combinations of friends can you invite to your house to watch the movie?

Write down two different ideas of how you can solve this problem:

1.

2.

Write down the rearrangement I put on the board:

SITUATION B

How would the result of this problem change if the friends were also picky about who sits on the left, right, and middle of the couch?

When we have **n** objects and we would like a group **k** of them in no particular order, we will call this a **combination**.

The formula for the number of groupings in this situation is:

When we have **n** objects and we would like an ordered group **k** of them, we will call this an **ordered selection**.

The formula for the number of groupings in this situation is:

There is a very large number of problems that we can solve with these two powerful ideas. For example:

1. A deck of cards has 52 different cards in 4 suits, and 13 different values for the . How many hands of five cards can be drawn?

2. Suppose that you want to draw a hand of cards that has two cards with the same value and three other cards with a different value. How many different hands are there?

3. Suppose that someone has **4** different pieces of fruit and is going to give out those pieces of fruit to **10** different children. If each kid can receive at most one piece of fruit (so that some kids don't get any), how many different ways can they be given out?

In the below problems, letters stand for whole numbers.

4. Suppose that we want to find how many different ways we can write x = 5. How many ways can this be done?

5. Suppose that we want to find how many different ways we can write x + y = 5. How many ways can this be done?

6. Suppose we want to find how many different ways we can write x + y + z = 5. How many ways can this be done?

7. Suppose we want to find how many different ways we can write x + y + z + q = 5. How many ways can this be done?

8. Suppose we want to find how many different ways we can write x + y + z + q + w = 5. How many ways can this be done?

Add up the results from problems 4, 5, 6, 7, 8:

In the fall, we had an explanation for this strange result. Let's review it below:

HOMEWORK: Show that $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$. Try to do it in two different ways.