Properties of the Number Seven[1]

Start by solving the following place-value problem:

A six-digit number having 1 as its leftmost digit becomes three times bigger if we take this digit off and put it and the end of the number. What is this number?

This information will come in useful later. For now, we are going to find some decimal representations of fractions whose denominator is 7.

Below, divide 100 …. 0000 (20 zeroes) by 7.

Do you see a pattern? What is it?

How could you have saved yourself work?

[1] This handout was made with the use of some problems from *Algebra*, by Gelfand & Shen.

Now, on a piece of scratch paper, divide 20 … 000 (20 zeroes), 30 … 000 (20 zeroes), 40 … 000 (20 zeroes), etc., by 7, and write your results in the table below.

10 … 000 (20 zeroes) ÷ 7	
20 … 000 (20 zeroes) ÷ 7	
30 … 000 (20 zeroes) ÷ 7	
40 … 000 (20 zeroes) ÷ 7	
50 … 000 (20 zeroes) ÷ 7	
60 … 000 (20 zeroes) ÷ 7	
70 … 000 (20 zeroes) ÷ 7	

Therefore, the decimal representation of each fraction is as below:

1 ÷ 7	
2 ÷ 7	
3 ÷ 7	
4 ÷ 7	
5 ÷ 7	
6 ÷ 7	
7 ÷ 7	

Now, on a separate sheet of paper, multiply 142857 by 1, 2, 3, 4, 5, 6, and 7, and look at the results.

142857 x 1	
142857 x 2	
142857 x 3	
142857 x 4	

142857 x 5	
142857 x 6	
142857 x 7	

Note that if we add fractions $\frac{m}{7} + \frac{n}{7}$, we get another fraction whose denominator is 7. How do we find the numerator?

If we multiply fractions $\frac{m}{7} * \frac{n}{7}$, we get another fraction whose denominator is 7. How do we find the numerator?

Homework: What kind of decimal do we get when we divide a whole number by 2 or 5?

Try these exercises with another prime number than 7 and see what happens.