GEOMETRIC PROBABILITY

You have already learned a fair amount of *discrete probability*, which is based on counting (sometimes fancy counting). Over the next couple weeks we will explore some *geometric probability* problems, which require us to be able to compute lengths, areas, and volumes of various geometric objects.

If a figure is not given, start by trying to make a drawing which will help solve the problem. For all problems, give exact answers. If you would like to approximate using a calculator, it's fine to do that as an extra step.

Most of these problems are gathered from Alcumus at www.artofproblemsolving.com. You can do more on your own by setting your focus to Counting & Probability → Using Geometry in Probability.

Problems:

1. One number x is selected at random from the interval $(0, 5)$. What is the probability that a triangle with sides of length 1, 2, and x exists?

2. A traffic light runs repeatedly through the following cycle: green for 30 seconds, then yellow for 3 seconds, and then red for 30 seconds. Leah picks a random three-second time interval to watch the light. What is the probability that the color changes while she is watching?

3. Suppose you have two random number generators. Each will give you a random real number between 1 and 6. (For example, the two numbers might be $\sqrt{3}$ and 4.5789123.) What is the probability that the sum of your two numbers is greater than 9?

4. Inside a square of side length 5cm, you choose a point P at random. If $\mathcal{O}$ is the center of the square, what is the probability that the length $P\mathcal{O}$ is less than 3?

5. A point (x, y) is randomly selected such that $0 \le x \le 8$ and $0 \le y \le 4$. What is the probability that $x + y \le 4$? Express your answer as a common fraction.

6. A point P is randomly selected from the square region with vertices at $(\pm 2, \pm 2)$. What is the probability that P is within one unit of the origin?

7. Point (x, y) is randomly picked from the rectangular region with vertices at $(0, 0), (2008, 0), (2008, 2009)$, and $(0, 2009)$. What is the probability that $x > 2y$?

8. Alice and Bob both go to a party which starts at 5:00. Each of them arrives at a random time between 5:00 and 6:00. What is the probability that the number of minutes Alice is late for the party plus the number of minutes Bob is late for the party is less than 45?

9. Annie and Xenas each arrive at a party at a random time between 2:00 and 4:00. Each stays for 45 minutes and then leaves. What is the probability that Annie and Xenas see each other at the party?

10. A point with coordinates $(x,\ y)$ is randomly selected such that $0 \le x \le 10$ and $0 \le y \le 10$. What is the probability that the coordinates of the point will satisfy $2x + 5y \ge 20$?

11. A point (x, y) is randomly selected such that $0 \le x \le 3$ and $0 \le y \le 6$. What is the probability that $x + y \le 4$?

12. A point (x, y) is randomly and uniformly chosen inside the square with vertices (0,0), (0,2), (2,2), and (2,0). What is the probability that $x + y < 3$?

13. Select numbers a and b between 0 and 1 independently and at random, and let c be their sum. Let A, B, and C be the results when a, b, and c, respectively, are rounded to the nearest integer. What is the probability that $A + B = C$?

14. A point P is randomly placed in the interior of the right triangle below. What is the probability that the area of triangle PBC is less than half of the area of triangle ABC?

15. In triangle ABC, $AB = 5$, $BC = 4$, and $CA = 3$. Point P is randomly selected inside triangle ABC. What is the probability that P is closer to C than it is to either A or B?

16. Two numbers, x and y are selected at random from the interval $(0, 3)$. What is the probability that a triangle with sides of length 1, x, and y exists?

17. Two numbers, x and y are selected at random from the interval $(0, 5)$. What is the probability that a triangle with sides of length 2, x, and y exists?

18. You have a stick that is 10cm long. Two different points on the wire are selected at random (not the endpoints), and you snip the stick at those two points. You now have three smaller sticks. (Assume no length is lost in the cutting process, so the 3 small lengths add to 10cm.) What is the probability that your three small sticks can form a triangle?

19. John needs to catch a train. The train arrives randomly some time between 2:00 and 3:00, waits for 20 minutes, and then leaves. If John also arrives randomly between 2:00 and 3:00, what is the probability that the train will be there when John arrives?

20. Rectangle $ABCD$ has center O and $AB/AD = k$. A point is randomly chosen from the interior of rectangle $ABCD$. What is the probability that it is closer to O than to any of the four vertices?

21. A point in space (x, y, z) is randomly selected so that $-1 \leq x \leq 1, -1 \leq y \leq 1, -1 \leq z \leq 1$. What is the probability that $x^2 + y^2 + z^2 \leq 1$?