ADVENTURES IN CUTTING PAPER

Today we'll explore a couple cool areas of math which feature cutting paper in interesting ways. First we'll take a look at cylinders and Möbius strips and generalizations with more half-twists. What happens when we carefully cut them up? Then we'll start to explore the Fold and Cut Theorem developed by Erik Demaine and various collaborators. We start with some relatively easy shapes we can make with a single straight cut, including some ocean creatures, and if there is time, we'll talk a little about fancier algorithms.

Next week, please bring scissors, as we will have yet another cool paper-cutting activity, and this one will work best if everyone is able to work on their own project.

Möbius Explorations:

1. Start by drawing an annulus – this is a closed disk (the boundary of a circle plus its interior region), with the interior of a smaller circle removed. It should look like a 2-d donut. If you cut one from paper, it would have two sides – a front and a back. How many boundary edges does it have? Here, the boundary will be made of curves, no necessarily straight line segments.

2. Next sketch a cylinder here. How many sides does it have? How many edges on its boundary?

3. Next, let's consider a Möbius strip. Much harder to draw on 2-d paper. How many sides does it have, and how many boundary edges? This is hard to visualize in your mind, so I'll give you a Möbius strip to hold and draw on.

4. What do you predict will happen if we cut a Möbius strip down the center?

5. What if instead of cutting down the center, we drew and cut out a curve staying about a third of the width away from an edge? (The one third does not need to be exact, but it should be less than halfway across the width.)

6. What else can we try?

Fold and Cut – as much folding as you like and then one single straight cut all the way across

1. Let's start with two relatively simple shapes. Can you figure out how to make a square with a single straight cut? You should also be left with a square shaped hole in your paper. Take notes on what folding to do and where to cut.

2. How about a triangle? Does the difficulty change depending on whether the triangle is equilateral, isosceles, or scalene?

3. Let's try more (approximately) regular polygons. Is a $2n$-gon (even number of sides) easier than a $(2n+1)$-gon (odd number of sides), or are they of roughly equal difficulty. See if you can figure out how to get 5, 6, 8, or 9 sides. (I skipped 7 on purpose since 360 is not divisible by 7. It's still doable, but you'll need fractions for the angles.)

4. How about ocean creatures? Can we make a one-cut starfish? A one-cut fish? Super extra credit – a one-cut seahorse?

5. Your homework is to draw a few moderately difficult (for you) polygonal shapes and figure out how to fold and cut them. If you are creatively stuck, you can try letters of the alphabet.