FUSING DOTS / EXPLODING DOTS

Over the next few weeks, we will explore systems of *fusing dots machines*. These are extremely cool for two big reasons – first, they can help us understand standard arithmetic algorithms in base 10, our usual number system, and second, they can introduce us to some exotic base systems.

Let's first explore the $\boxed{1 \leftrightarrow 2}$ machine. For all of our machines, we'll have a line of boxes, with one box designated the units box. To build representations of the integers, we always input dots in the units box. For the $\boxed{1 \leftrightarrow 2}$ machine, we have an additional rule – whenever two dots are in the same box, they may fuse together into a single dot one box to the left. In figures, this looks like: $\boxed{\bullet}\boxed{} = \boxed{}\boxed{\bullet\bullet}$. Let's build representations for the first few integers together. As we build bigger integers, we add more boxes to the left as necessary. For the $\boxed{1 \leftrightarrow 2}$ machine, we will say that a representation is in simplified form if every box contains at most one dot.

1: $\boxed{\bullet}$

2: $\boxed{\bullet\bullet} = \boxed{\bullet}\boxed{}$

3: $\boxed{\bullet\bullet\bullet} = \boxed{\bullet}\boxed{\bullet}$

4: $\boxed{\bullet\bullet\bullet\bullet} = \boxed{\bullet}\boxed{\bullet\bullet} = \boxed{\bullet\bullet}\boxed{} = \boxed{\bullet}\boxed{}\boxed{}$

Mathematicians like to be efficient with our writing, so perhaps we can just record the number of dots left in each box after simplification.

n	$\boxed{1 \leftrightarrow 2}$ machine representation
1	1
2	10
3	11
4	100
5	
6	
7	
8	
9	
10	

n	$\boxed{1 \leftrightarrow 2}$ machine representation
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

Questions to think about for the $\boxed{1 \leftrightarrow 2}$ machine (add your own questions too!):

- Does this look like a familiar number system?
- When must we add new boxes, or equivalently, new digits in our shorthand representations?
- Is this a base system?
- Is there a faster way to find an integer's representation, other than building it one dot at a time?
- Can we add integers using the fusing dots machine?
- Can we subtract integers using the fusing dots machine?
- Can we multiply integers using the fusing dots machine?
- Can we divide integers using the fusing dots machine?
- Does our machine only make sense for positive integers, or can we use it for other numbers as well (e.g. negative numbers, rational numbers)?
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Now, what other machines could we explore? List some ideas below. Do they all make sense?

$\boxed{1 \leftrightarrow 3}$

Rules for the $\boxed{1 \leftrightarrow 3}$ machine:

n	$\boxed{1 \leftrightarrow 3}$ machine representation
1	1
2	2
3	10
4	11
5	
6	
7	
8	
9	
10	

n	$\boxed{1 \leftrightarrow 3}$ machine representation
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

After you have some data in your table, think more about the questions on page 2, but for this fusing dots machine instead.

Rules for the $\boxed{2 \leftrightarrow 3}$ machine:

n	$\boxed{2 \leftrightarrow 3}$ machine representation
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

n	$\boxed{2 \leftrightarrow 3}$ machine representation
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

After you have some data in your table, think more about the questions on page 2, but for this fusing dots machine instead.

Rules for the ☐ machine:

n	☐ machine representation
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

n	☐ machine representation
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

After you have some data in your table, think more about the questions on page 2, but for this fusing dots machine instead.

Rules for the ________ machine:

n	________ machine representation
1	1
2	2
3	20
4	21
5	
6	
7	
8	
9	
10	

n	________ machine representation
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

After you have some data in your table, think more about the questions on page 2, but for this fusing dots machine instead.