Part 1: Find the Expected Value of each of the following games.

1. Flip a single fair coin, and win one dollar if the flip is heads.

2. Roll a single, fair, six-sided die, and win three dollars if you roll a 1, but no money otherwise.

3. Flip two fair coins, and win one dollar for each heads.

4. Roll a single, fair, six-sided die, and win the amount shown.

5. Roll two fair six-sided dice, and win the sum of the two rolls.

6. Roll three fair four-sided dice, and win the sum of the three rolls.

7. Flip three coins, and win the square of the total number of heads. Can you generalize to N coins?

For the following games, you are given a deck of 40 cards. The cards are numbered 1-10, and there are four cards for each number. Additionally, half the cards for a given value are red, and the other half are black. Find the expected value of each game.

8. A single card is drawn from the deck, you win the amount shown.

9. Two cards are drawn from the deck without replacement. You win the total amount shown.

10. Five cards are drawn from the deck without replacement. You win five dollars for every 5 which is drawn.

11. Four cards are drawn from the deck without replacement. You win the total amount showing on red cards.

12. Four cards are drawn from the deck without replacement. You win the *square* of the number of red cards drawn.

13. Two cards are drawn from the deck without replacement. You win the total amount if both cards are red, and nothing otherwise.

14. Two cards are drawn from the deck without replacement. You win the total amount if both cards are greater than 5, and nothing otherwise.

Part 2: Possibly Infinite Games

Find the expected value of each game.

15. You flip a fair coin until you flip a tails, and win a dollar for each flip.

16. You roll a twenty-sided die until you roll a 20, and win a dollar for each roll.

17. You start with one dollar, and flip a fair coin. If it is heads, double your money. If it is tails, you're done. Repeat this until you get a tails.

18. You roll a twenty-sided die until you roll two twenties, and win a dollar for each roll.

19. You have a deck with 10 cards numbered 1-10. You draw a card, record the value, then shuffle it back into the deck. You play until you've drawn every card at least once, and win a dollar for each draw.

20. You flip a fair coin until you flip a tails, and win the square of the total number of flips.

21. *Challenge* You flip a fair coin until you've flipped one more heads than tails, and win a dollar for each flip.