Two-Square Nim 10-10

Put **10 tokens** in square A and **10 tokens** in square B. On your turn, you can take any number of tokens from **one and only one** of the two squares. The person to take the **last** token in the entire game wins.

1) Does player 1 or player 2 have the advantage in this game? (circle one) **P1** **P2**

Two-Square Nim 7-13

Put **7 tokens** in square A and **13 tokens** in square B and play with the same rules as before.

2) Does player 1 or player 2 have the advantage in this game? (circle one) **P1** **P2**

3) What strategy can they use to win every game?

4) Which Two-Square Nim games (X-Y) that can be won with this strategy?

Three-Square Nim 7-7-6

Put **7 tokens** in square A, **7 tokens** in square B, and **6 tokens** in square C. On your turn, you can take any number of tokens from **one and only one** of the three squares. The person to take the **last** token in the entire game wins.

5) Does player 1 or player 2 have the advantage in this game? (circle one) **P1** **P2**

6) What strategy can they use to win every game?

Three-Square Nim (?-?-?)

Design your own 3-square Nim game by deciding on the starting distribution of tokens.

7) What distribution did you choose? _____ - _____ - _____

8) Does player 1 or player 2 have the advantage in your game? (circle 1) **P1** **P2**

9) What strategy can they use to win every game?

ALL 3-Square Nim Games

Here is a list of all of the games that start with 20 stones total:

10) Convince yourself that the chart below includes every game with 20 total stones.

11) Figure out which player has the advantage for as many games as you can.

12) What pattern(s) do you see and what strategies do the players need to know?

COLOR KEY ☐ = Player 1 Wins ☐ = Player 2 Wins

20-0-0						
19-1-0						
18-2-0	18-1-1					
17-3-0	17-2-1					
16-4-0	16-3-1	16-2-2				
15-5-0	15-4-1	15-3-2				
14-6-0	14-5-1	14-4-2	14-3-3			
13-7-0	13-6-1	13-5-2	13-4-3			
12-8-0	12-7-1	12-6-2	12-5-3	12-4-4		
11-9-0	11-8-1	11-7-2	11-6-3	11-5-4		
10-10-0	10-9-1	10-8-2	10-7-3	10-6-4	10-5-5	
		9-9-2	9-8-3	9-7-4	9-6-5	
				8-8-4	8-7-5	8-6-6
						7-7-6

Name: _____

Notes – BMC Math Circle – How to Win Every Game – 9/1/15

Binary – Base 2

What if you didn't want to teach 10 symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) in school, but instead, only 2 (0, and 1)? Make a list, in increasing order, of all of the numbers that only contain the digits 0 and 1?

0:	0	8:	16:	SPICY CHALLENGE:
1:	1	9:	17:	What do you think the
2:	10	10:	18:	100^{th} such number will be?
3:	11	11:	18:	
4:	100	12:	20:	…
5:		13:	21:	100:
6:		14:	22:	101:
7:		15:	23:	102:

Can we explain these numbers in such a way that 100 *means* 4, 1000 *means* 8, and 10011 *means* 19?

Powers

	0	1	2	3	4	5	6	7
Base 10								
Base 2								

# in base 10	Same # in base 2 (0 and 1 are the only digits)						
	128s	64s	16s	8s	4s	2s	1s .
2	___	___	___	___	___	___	___
20	___	___	___	___	___	___	___
3	___	___	___	___	___	___	___
8	___	___	___	___	___	___	___
16	___	___	___	___	___	___	___
23	___	___	___	___	___	___	___
64	___	___	___	___	___	___	___
63	___	___	___	___	___	___	___
127	___	___	___	___	___	___	___

Nim in Binary

20 – 10100 0 – 00000 0 – 00000						
19 – 10011 1 – 00001 0 – 00000						
18 – 10010 2 – 00010 0 – 00000	18 – 10010 1 – 00001 1 – 00001					
17 – 10001 3 – 00011 0 – 00000	17 – 10001 2 – 00010 1 – 00001					
16 – 10000 4 – 00100 0 – 00000	16 – 10000 3 – 00011 1 – 00001	16 – 10000 2 – 00010 2 – 00010				
15 – 01111 5 – 00101 0 – 00000	15 – 01111 4 – 00100 1 – 00001	15 – 01111 3 – 00011 2 – 00010				
14 – 01110 6 – 00110 0 – 00000	14 – 01110 5 – 00101 1 – 00001	14 – 01110 4 – 00100 2 – 00010	14 – 01110 3 – 00011 3 – 00011			
13 – 01101 7 – 00111 0 – 00000	13 – 01101 6 – 00110 1 – 00001	13 – 01101 5 – 00101 2 – 00010	13 – 01101 4 – 00100 3 – 00011			
12 – 01100 8 – 01000 0 – 00000	12 – 01100 7 – 00111 1 – 00001	12 – 01100 6 – 00110 2 – 00010	12 – 01100 5 – 00101 3 – 00011	12 – 01100 4 – 00100 4 – 00100		
11 – 01011 9 – 01001 0 – 00000	11 – 01011 8 – 01000 1 – 00001	11 – 01011 7 – 00111 2 – 00010	11 – 01011 6 – 00110 3 – 00011	11 – 01011 5 – 00101 4 – 00100		
10 – 01010 10 – 01010 0 – 00000	10 – 01010 9 – 01001 1 – 00001	10 – 1010 8 – 01000 2 – 00010	10 – 01010 7 – 00111 3 – 00011	10 – 01010 6 – 00110 4 – 00100	10 – 01010 5 – 00101 5 – 00101	
		9 – 01001 9 – 01001 2 – 00010	9 – 01001 8 – 01000 3 – 00011	9 – 01001 7 – 00111 4 – 00100	9 – 01001 6 – 00110 5 – 00101	
				8 – 01000 8 – 01000 4 – 00100	8 – 01000 7 – 00111 5 – 00101	8 – 01000 6 – 00110 6 – 00110
						7 – 00111 7 – 00111 6 – 00110

Solving Nim

The Nim-Sum of a game is 0 if:

The games that can be won by player 1 are:

The games that can be won by player 2 are:

An Invariant is…

Intuitive definition (describe it in simple word):

Rigorous definition (the precise mathematical definition):

Sneak preview of next week…
"Magic with a Half-Deck" -- Base 3

What if we allow 3 numbers in our system: 0, 1, and 2

0:	5:	10:	15:	20:	25:	Can you make
1:	6:	11:	16:	21:	26:	the same kind
2:	7:	12:	17:	22:	27:	of meaning
3:	8:	13:	18:	23:	28:	here as we did
4:	9:	14:	19:	24:	29:	for base 2?

NAME(S): _____ DATE: _____

CHANGE THE GAME – NIM VARIANTS

NEW GAME NAME:

RULES	STRATEGY
This game is like Nim with ____ squares *At the beginning of the game, the stones are...* *The rules are like Nim except...* *The winner is the player who...*	

ADDITIONAL THOUGHTS:

FUN RATING: (NOT FUN) 0 1 2 3 4 5 (VERY FUN)
EXPLAIN:

DIFFICULTY RATING: (NOT FUN) 0 1 2 3 4 5 (VERY FUN)
EXPLAIN: