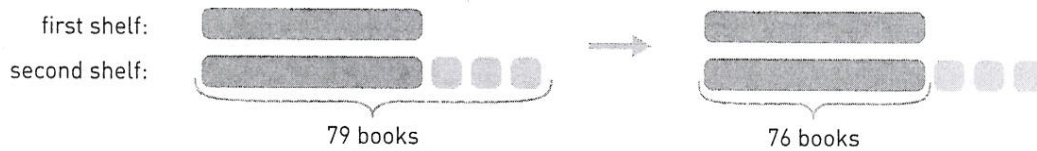


BMC Worksheet: Word Problems and Common Sense.

“Draw a Picture — Solve the Problem!”

Problem 1. 79 books are placed on 2 shelves. One shelf has 3 more books than the other. How many books are there on each shelf?

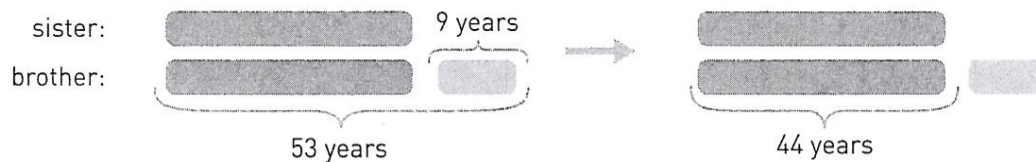
Discussion:



Since the two rows are equal, each has $76 \div 2 = 38$ books. Therefore, originally there were 38 books on one shelf and 41 books on the other.

Problem 2. The ages of two siblings add up to 53 years. The brother is 9 years older than the sister. How old is the brother?

Discussion:



The common parts of both numbers add up to $53 - 9 = 44$. Therefore, the common part is $44 \div 2 = 22$. Thus, the brother's age is $22 + 9 = 31$.

Problem 3. Three consecutive numbers add up to 165. What are these numbers?

Discussion:



Together, they add up to $165 - 3 = 162$; thus each piece is 54 units long, and the numbers are 54, 55, 56.

Problem 4. A mountain gnome has a collection of precious stones. His dream is to have three times as many stones as he has now. The gnome noticed that his dream will come true if he adds 18 stones to his collection. How many precious stones does he have now?

Make your own picture:

Problem 5. Max and Bella spent the same amount of money at the amusement park. Max went on 3 roller coaster rides and 1 merry-go-round ride, while Bella rode the roller coaster twice and the merry-go-round 4 times. How many times more expensive is a roller coaster ticket compared to a merry-go-round ticket?

Make your own picture:

Working Together

Problem 6. Max eats up a jar of strawberry jam in 15 days, and Bella does the same in 10 days. How long would a jar of jam last if both Max and Bella ate from it?

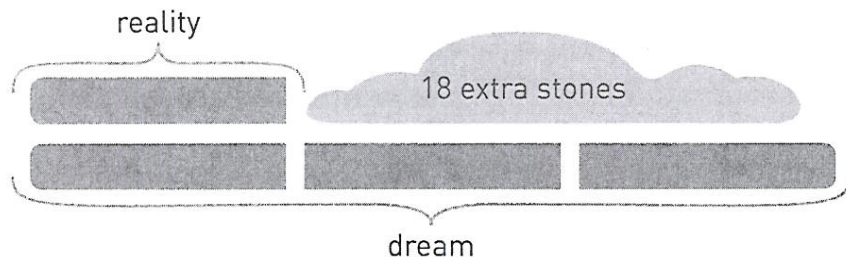
- two new terms:
 - *common multiple*
 - the *least common multiple*

Problem 7. Last year, it took 3 weeks for Grandma Ethel to knit blue scarves as gifts for all of grandchildren. This year, she wants to give them red scarves, and her daughter Mary will be helping her with knitting. It would take Mary 11 weeks to knit the complete set of scarves if she works on her own. How long will it take for the two of them to finish the job if they work together?

Problem 8. It takes 2 days for a horse to finish 1 bale of hay, 3 days for a cow to eat the same amount of hay, and 6 days for a goat to do the same. In how many days would the three of them together consume 1 bale of hay?

Problem 9. It takes 12 months for Santa Claus to prepare all the gifts for the upcoming Christmas. It would take 20 months for Santa's apprentice to do the same job. If they work together, how long will it take for them to prepare all the gifts?

Problem 4 Discussion.



If the gnome's dream comes true, he will be three times richer. So we make the bottom rectangle three times longer than the top one. Their difference is equal to twice the top rectangle, and we know that this difference equals 18. Therefore, the length of the top rectangle is 9.

Problem 5 Discussion. Here is the relevant picture:



Dark rectangles stand for the price of the roller coaster rides; the lighter ones stand for the merry-go-round rides. It follows from the picture that 1 roller coaster ride costs as much as 3 merry-go-round rides.

Problem 6 Discussion. Let's start by computing how many jars of jam each child would finish in 30 days. That's easy: 2 jars for Max and 3 jars for Bella. So Max and Bella together would eat 5 jars of jam in 30 days. Given this, we can easily calculate how long it would take for the two of them to finish one jar: $30/5 = 6$ days.

Problem 7 Discussion. First, let's find the least common multiple of both 3 and 11. That is 33. In 33 weeks, grandma will knit 11 complete sets of scarves, and Mary will knit 3 sets of scarves. Thus, together they will knit 14 sets of scarves in 33 weeks. Therefore, Grandma and Mary will make 1 set in $33/14$ weeks, which is 2 weeks and 5 days.

Problem 8: 1 day

Problem 9: $\frac{15}{2} = 7.5$ months



Problem 10. Jim, Bob, Tom, and Siegfried are four hungry trolls who took part in a rock-eating contest. Jim gobbled down 1 less rock than Bob; Bob ate 1 less than Tom, and Tom 1 less than Siegfried. How many rocks did each troll eat if it is known that together they consumed 2010 rocks?

Make a diagram:

Problem 11. Jeannie and Max had the same number of cookies. Max gave Jeannie 2 of his cookies.

(a) How many more cookies than Max does Jeannie have now?

(b) If Max now has only half as many cookies as Jeannie, how many cookies did each of them have in the beginning?

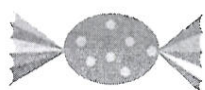
Problem 12. Old MacDonald has 32 animals on his farm—all pigs and hens. Together, these animals have 138 legs. How many pigs and how many hens are there at the farm?

More Practice (and Homework) Problems

Problem 1. (a) Connect the four dots below (left diagram) with three straight lines without lifting your pen off the paper, and finishing at the same point where you started.



(b) Connect the nine dots above (right diagram) with four straight lines without lifting your pen off the paper. In this problem you are not required to finish at the same point where you started.



Problem 2. The day after Halloween, Tim and Alex were admitted to the hospital with stomach cramps. Their mother told the emergency room doctor that she had found 141 empty candy wrappers in the boys' room.

The hospital policy is to apply a stomach pump to anyone who ate more than 65 candies. Tim claims that he does not need the procedure since he ate 7 less pieces of candy than Alex. Is he going to get his stomach pumped? How about Alex?

Problem 3. At the Magic Cookies and Candy Shop, a cookie costs twice as much as a piece of candy. Tim bought six cookies and three pieces of candy. Allie bought three cookies and six pieces of candy. Tim paid \$1.80 more than Allie. What are the prices of cookies and candy?

Problem 4. While visiting the Knights and Liars Island, you meet five local girls: Alice, Britney, Cindy, Debby and Elizabeth.

– Alice says that if you add two even numbers, you always get an even number.

– Britney declares that it is possible to find two odd numbers that add up to an even number.

– Cindy announces that the sum of two even numbers can never be odd.

– Debby states that all the girls in the group are knights.

– Elizabeth claims that if you add two odd numbers, you always get an odd number.

Find out which of these girls are liars and which are knights. Justify your answer.

Problem 5. Brenda the Beaver stumbled upon a pile of long logs left in a logger's camp. She decided to build a dam from these logs. For several days, Brenda worked on cutting the long logs into smaller pieces. By the time Brenda was out of long logs, she had made 83 cuts, and there were 109 short logs. How many long logs were there at the start?

Problem 6. 25 students study at the Prime Factor Math Circle. Out of any 10 of them, at least one is a girl. Out of any 17 of them, at least one is a boy. How many girls and how many boys study at Prime Factor?

Problem 7. Three ponies and 1 gnome weigh as much as 2 ponies and 4 gnomes. How many times heavier is a pony than a gnome?

Problem 8. Alex wanted to add two numbers using his calculator. While entering the second number, he accidentally added an extra zero to the end of this number. As a result, his total turned out to be 7641 instead of 2331. What were the original numbers?

Problem 9. Fifteen big and small mafia bosses held a secret meeting. During this meeting, they smoked cigars. Every big boss smoked four more cigars than every small boss. When secret agent 00X sneaked into the meeting room, he collected 99 cigar butts. How many big bosses came to the meeting?

Problem 10. A gang of twenty outlaws stole 109 gold coins. They split the loot as follows: common outlaws each received 3 coins, and the ring leaders each received an unknown (identical) number of coins. How many ring leaders are there in the gang? List and justify all possible answers.

Selected Answers to BMC "Meet the Cube" (from Nov 17)

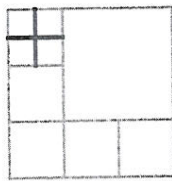
Problem 1 Solution.

- (a) 4 on the bottom, 3 on the rear, and 6 on the left face.
 (b) $\underset{4}{2}$ on the bottom, $\underset{2}{5}$ on the rear, and $\underset{5}{4}$ on the left face.

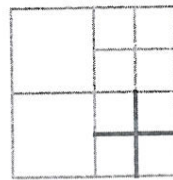
Problem 2 Solution. A hexagon.

Problem 3 Solution. This triangle is special because all of its sides are the diagonals of the faces of the cube. Since all faces of the cube are squares of the same size, all these diagonals are of the same length. Thus, all the sides of the triangle are equal, and all the angles are equal as well. Therefore, the angles measure 60 degrees each.

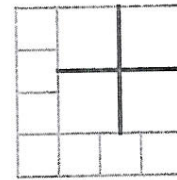
Problem 5 Solution.



9



10



11

Problem 6 Solution. The key to this problem is that every time a dwarf cuts a square into four smaller squares, the total number of pieces increases by three (one bigger square is replaced by four smaller ones). Therefore, no matter how the dwarfs choose the pieces to cut, the total number of pieces after seven cuts will be $4 + 7 \times 3 = 25$.

Problem 7 Solution. (a) Yes. Jumping right twice and left once puts the cricket 1 inch to the right of the starting point. Jumping 3 times right and twice left puts the cricket 1 inch to the left of the starting point.

(b) Let's use the same approach as in Session 10 (Alternations).

Observe that all possible landing points of the cricket are located at 1-meter intervals. Paint each of these points red or blue in such a way that the points of different colors alternate. Since the cricket's jumps are 3 or 5 meters long, after every leap he lands on a point of a different color. Thus, the colors of his landing points alternate. Now let's assume that the cricket's starting point is red. Then he'll land on a red-colored point after the second jump and after any even number of jumps. However, the point one inch to the right from the starting point is blue. Therefore, there is no way for the cricket to end up at this point after an even number of jumps.

(c) After 23 jumps, the cricket will end up on a blue point. However, the point 10 inches to the left is colored red. Therefore, there is no way for the cricket to end up at this point after 23 jumps.