Your Name: _____

WARMUPS

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WARMUPS:

(1) ARITHMETIC

PROOF:

Which of these is NOT a property of all even and odd numbers?

- even + even = even
- odd + odd = even
- even × even = even
- odd × odd = even
- even × odd = even

(2) PROBABILITY

EXPLAIN:

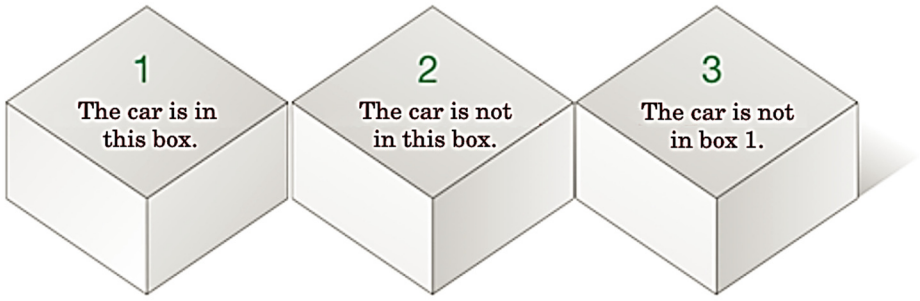
You are a prisoner about to be executed by a gunman. As you move into position the gunman takes a six-chamber revolver and places inside two bullets next to one another. He spins the chamber, locks it in place, aims and fires... and nothing happens. At this point he tells you he will fire once more and if you are still standing afterwards you are free to go.

He gives you two options- either fire again, or spin the chamber and then fire. What do you do?

- Spin then fire
- Just fire
- It doesn't matter

(3) LOGIC

EXPLAIN:



If only one of these 3 statements is true, which box has the car?

(4) What's the difference between **a mathematical proof** and **a convincing argument?**

3 WAYS TO "PROVE" THAT 1=2

PROOF 1: 2 = 1

Circle the step that's wrong. What's wrong with it?

"Proof": $2 = 1$

$-2 = -2$
$4 - 6 = 1 - 3$
$4 - 6 + 9/4 = 1 - 3 + 9/4$
$(2 - 3/2)^2 = (1 - 3/2)^2$
$2 - 3/2 = 1 - 3/2$
$2 = 1$

PROOF 2: 1 = 2

Circle the step that's wrong. What's wrong with it?

Step 1: Let $a=b$.
Step 2: Then $a^2 = ab$,
Step 3: $a^2 + a^2 = a^2 + ab$,
Step 4: $2a^2 = a^2 + ab$,
Step 5: $2a^2 - 2ab = a^2 + ab - 2ab$,
Step 6: and $2a^2 - 2ab = a^2 - ab$.

Step 7: This can be written as $2(a^2 - ab) = 1(a^2 - ab)$,

Step 8: and cancelling the $(a^2 - ab)$ from both sides gives 1=2

PROOF 3: 1 + 1 = 1

Circle the step that's wrong. What's wrong with it?

$a = 1$
$b = 1$

$a = b$
$a^2 = b^2$
$a^2 - b^2 = 0$
$(a-b)(a+b) = 0$
$(a-b)(a+b)/(a-b) = 0/(a-b)$
$1(a+b) = 0$
$(a+b) = 0$
$1 + 1 = 0$
$2 = 0$
$1 = 0$
$1 + 1 = 1$

EXTRA SPICY I (uses complex numbers):

Circle the step that's wrong. What's wrong with it?

$$\sqrt{-1} = i \quad \dots(1)$$
$$\frac{1}{\sqrt{-1}} = \frac{1}{i} \quad \dots(2)$$
$$\frac{\sqrt{1}}{\sqrt{-1}} = \frac{1}{i} \quad \dots(3)$$
$$\sqrt{\frac{1}{-1}} = \frac{1}{i} \quad \dots(4)$$
$$\sqrt{\frac{-1}{1}} = \frac{1}{i} \quad \dots(5)$$
$$\frac{\sqrt{-1}}{1} = \frac{1}{i} \quad \dots(6)$$
$$i = \frac{1}{i} \quad \dots(7)$$
$$i^2 = 1 \quad \dots(8)$$
$$-1 = 1 \quad \dots(9)$$

EXTRA SPICY II (Uses infinite series):

Circle the step that's wrong. What's wrong with it?

$$1 = 1$$

$$1 = 1 + 0 + 0 + 0\dots$$

$$1 = 1 + 1 - 1 + 1 - 1 + 1 - 1\dots$$

$$1 = 2 - 1 + 1 - 1 + 1 - 1\dots$$

$$1 = 2 + 0 + 0\dots$$

$$1 = 2$$

EXTRA SPICY III (Uses some advanced algebra):

$$(n+1)^2 = n^2 + 2n + 1$$

Expansion:

$$(n+1)^2 - (2n+1) = n^2$$

Subtract from both sides:

$$(n+1)^2 - (2n+1) - n(2n+1) = n^2 - n(2n+1)$$

Add to both sides:

$$(n+1)^2 - (n+1)(2n+1) = n^2 - n(2n+1)$$

Factor:

$$(n+1)^2 - (n+1)(2n+1) + \frac{(2n+1)^2}{4} = n^2 - n(2n+1) + \frac{(2n+1)^2}{4}$$

Add to both sides:

$$\left[(n+1) - \frac{2n+1}{2}\right]^2 = \left[n - \frac{2n+1}{2}\right]^2$$

Square root:

$$(n+1) - \frac{2n+1}{2} = n - \frac{2n+1}{2}$$

Subtract from both sides:

$$n+1 = n$$

Subtract from both sides:

$$1 = 0$$

Circle the step that's wrong. What's wrong with it?