Number Theory Snacks
Berkeley Math Circle, Intermediate 2
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Warm-up problems
Please think about the following questions as we get started – especially #1.
If there is anything you have covered recently in BMC, we'll move on to other topics.

1) Primes and Factoring:
a) What is the general procedure to determine which numbers are primes? Make a list of them up to 30.

b) Which numbers from 200 to 210 are primes? Make sure you are as **lazy** as possible when answering this question: don't do any calculations you don't need to do!

c) Ok, that was nice. Now factor 8051. Please work hard to find a way you can answer this lazily.

d) Same question for 2041, but please find a more general way to be lazy.

e) How many primes are there? Way fewer than natural numbers? How would you describe this?

2) Truncated triangular numbers:
a) What is the formula for the nth triangular number: $1 + 2 + 3 + \ldots + n$?

b) $5 = 2 + 3$ and $12 = 3 + 4 + 5$ are examples of numbers that can be written as the sum of two or more consecutive natural numbers. Which natural numbers cannot be written this way?

3) Reducing the sum of two fractions
Let's say a fraction of integers is "reduced" if you cannot cancel out a common factor, such as for 2/3, but not for 4/6.
For which integer choices of a and b are the following reduced (possible answers might include none, all, some – careful: this might not be quite as easy as it looks!):

a) a/3 + b/6   b) a/20 + b/12   c) a/10 + b/12