Pushing the Envelope
Families of Curves in the Plane

Berkeley Math Circle

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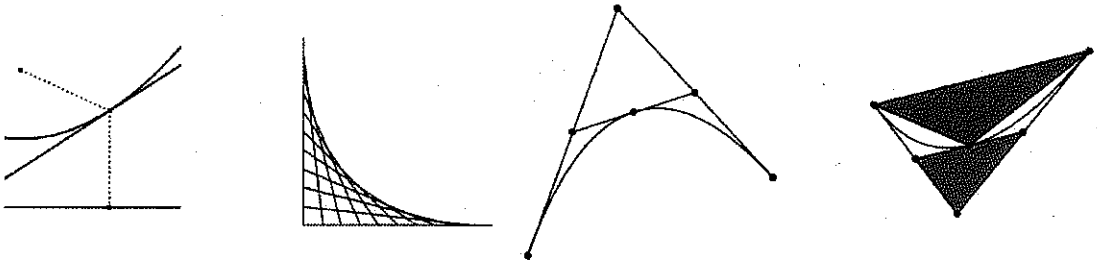
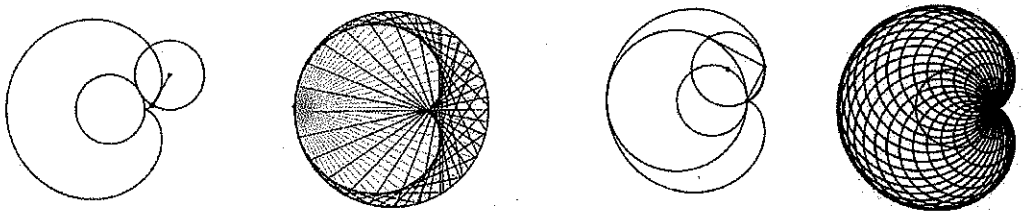
- **The Parabola as an Envelope of Lines**

The parabola (yes, there is only *one*!) is the **locus of points equidistant from a give point and a given line.** The line is called the directrix, and the point is called the focus.



A tangent to is the perpendicular bisector of the segment joining the point on the directrix to the focus.
Problems: the reflection property of a parabola: every ray coming from above the parabola and perpendicular to the directrix will be relected into the focus.

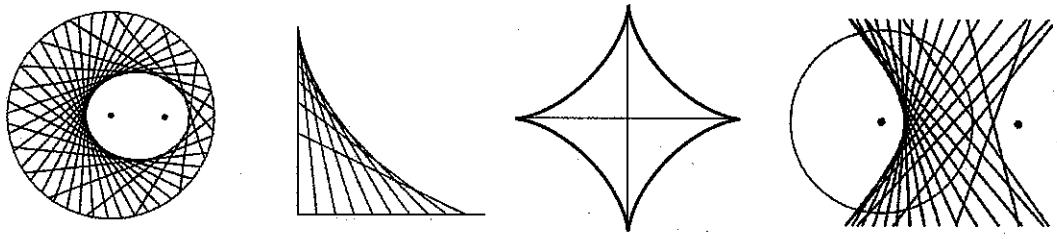
- **The Cardioid**

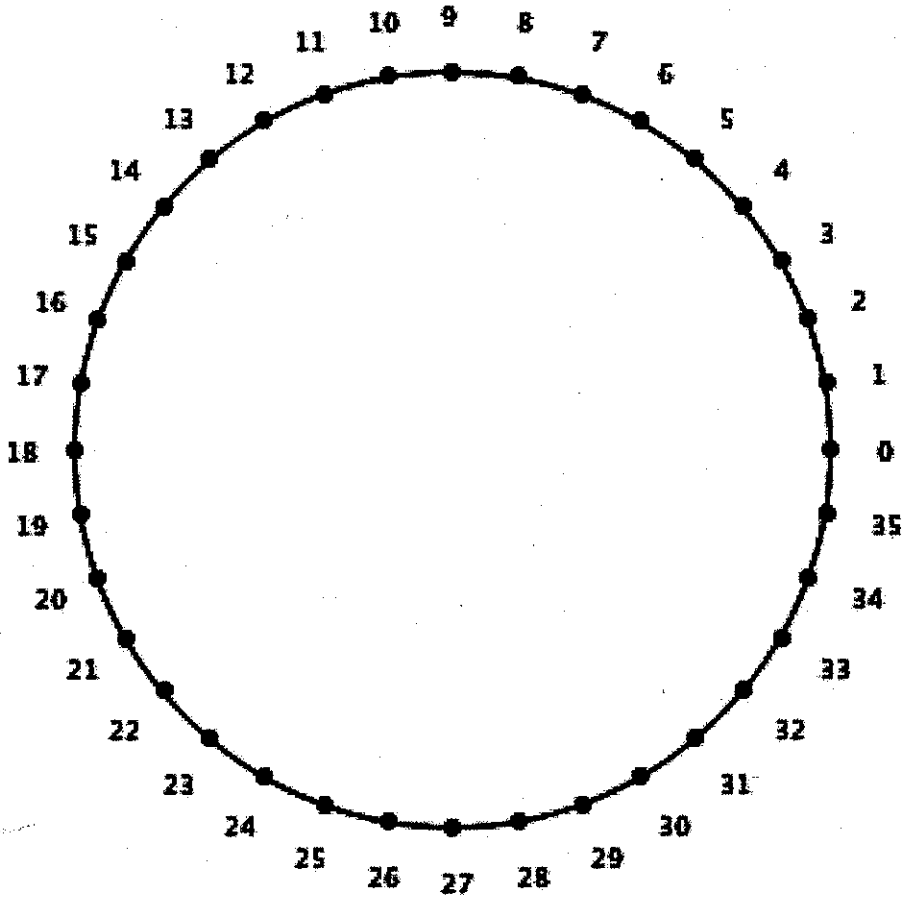


The path of a point on a circle which rolls around a congruent circle is a cardioid.
The cardioid is also the envelope of the family of reflected rays at the edge of the circle.
Finally the cycloid is the envelope of a family of circles.

- **Other Curves as Envelopes**

The ellipse The Astroid The Astroid The Hyperbola

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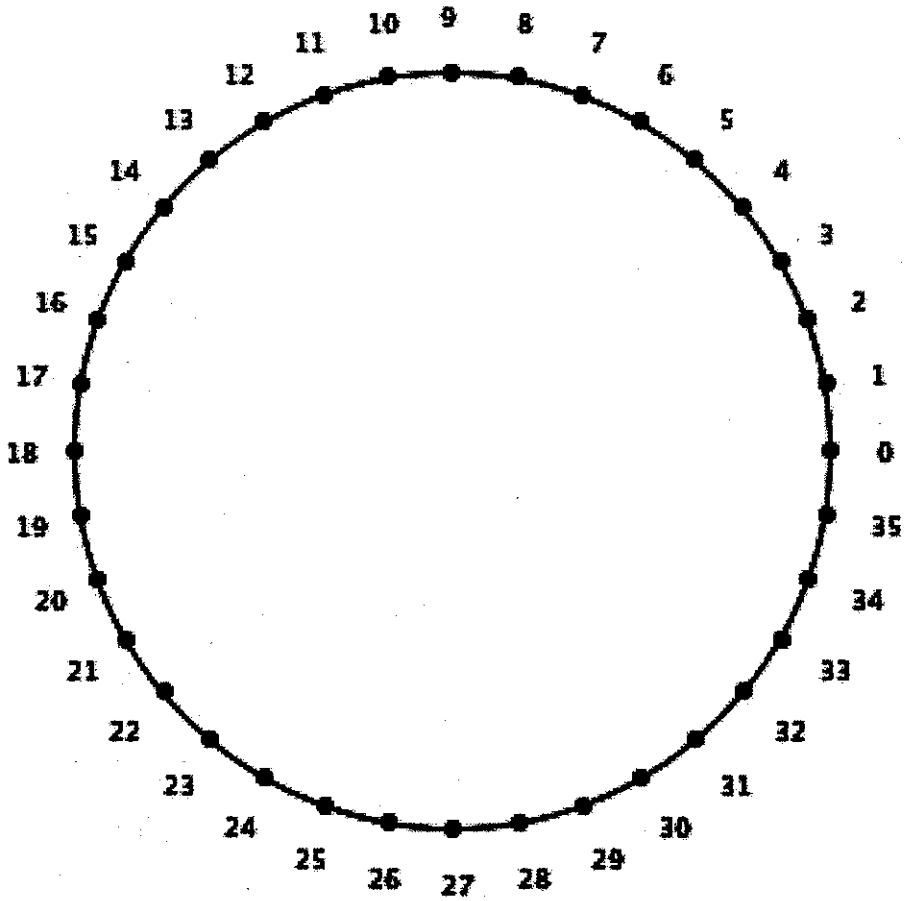
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