AMERICAN MATHEMATICS COMPETITIONS

2nd Annual American Mathematics Contest 10

AMC 10

TUESDAY, FEBRUARY 13, 2001

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question, multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** working time to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score in the top 1% on this AMC 10 will be invited to take the 19th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 27, 2001 or on Tuesday, April 10, 2001. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. The median of the list

$$n, n+3, n+4, n+5, n+6, n+8, n+10, n+12, n+15$$

is 10. What is the mean?

(A) 4 **(B)** 6 **(C)** 7 **(D)** 10 **(E)** 11

2. A number x is 2 more than the product of its reciprocal and its additive inverse. In which interval does the number lie?

(A) $-4 \leq x \leq -2$ **(B)** $-2 < x \leq 0$ **(C)** $0 < x \leq 2$

(D) $2 < x \leq 4$ **(E)** $4 < x \leq 6$

3. The sum of two numbers is S. Suppose 3 is added to each number and then each of the resulting numbers is doubled. What is the sum of the final two numbers?

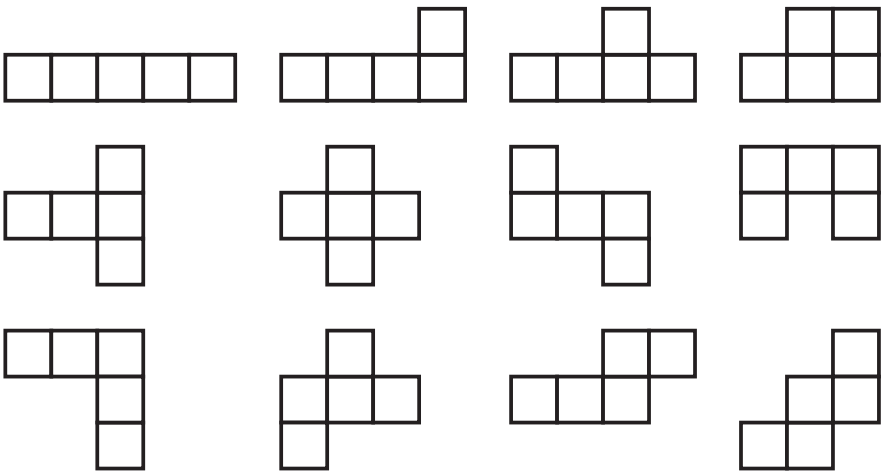
(A) $2S+3$ **(B)** $3S+2$ **(C)** $3S+6$ **(D)** $2S+6$ **(E)** $2S+12$

4. What is the maximum number for the possible points of intersection of a circle and a triangle?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

5. How many of the twelve pentominoes pictured below have at least one line of symmetry?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7



6. Let $P(n)$ and $S(n)$ denote the product and the sum, respectively, of the digits of the integer n. For example, $P(23) = 6$ and $S(23) = 5$. Suppose N is a two-digit number such that $N = P(N) + S(N)$. What is the units digit of N?

(A) 2 **(B)** 3 **(C)** 6 **(D)** 8 **(E)** 9

7. When the decimal point of a certain positive decimal number is moved four places to the right, the new number is four times the reciprocal of the original number. What is the original number?

(A) 0.0002 **(B)** 0.002 **(C)** 0.02 **(D)** 0.2 **(E)** 2

8. Wanda, Darren, Beatrice, and Chi are tutors in the school math lab. Their schedule is as follows: Darren works every third school day, Wanda works every fourth school day, Beatrice works every sixth school day, and Chi works every seventh school day. Today they are all working in the math lab. In how many school days from today will they next be together tutoring in the lab?

(A) 42 **(B)** 84 **(C)** 126 **(D)** 178 **(E)** 252

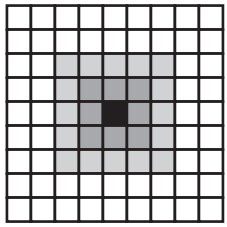
9. The state income tax where Kristin lives is levied at the rate of $p\%$ of the first \$28000 of annual income plus $(p + 2)\%$ of any amount above \$28000. Kristin noticed that the state income tax she paid amounted to $(p+0.25)\%$ of her annual income. What was her annual income?

(A) \$28000 **(B)** \$32000 **(C)** \$35000 **(D)** \$42000 **(E)** \$56000

10. If x, y, and z are positive with $xy = 24$, $xz = 48$, and $yz = 72$, then $x + y + z$ is

(A) 18 **(B)** 19 **(C)** 20 **(D)** 22 **(E)** 24

11. Consider the dark square in an array of unit squares, part of which is shown. The first ring of squares around this center square contains 8 unit squares. The second ring contains 16 unit squares. If we continue this process, the number of unit squares in the 100^{th} ring is



(A) 396 **(B)** 404 **(C)** 800 **(D)** 10,000 **(E)** 10,404

12. Suppose that n is the product of three consecutive integers and that n is divisible by 7. Which of the following is not necessarily a divisor of n?

(A) 6 **(B)** 14 **(C)** 21 **(D)** 28 **(E)** 42

13. A telephone number has the form $ABC-DEF-GHIJ$, where each letter represents a different digit. The digits in each part of the number are in decreasing order; that is, $A > B > C$, $D > E > F$, and $G > H > I > J$. Furthermore, D, E, and F are consecutive even digits; G, H, I, and J are consecutive odd digits; and $A + B + C = 9$. Find A.

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

14. A charity sells 140 benefit tickets for a total of \$2001. Some tickets sell for full price (a whole dollar amount), and the rest sell for half price. How much money is raised by the full-price tickets?

(A) \$782 **(B)** \$986 **(C)** \$1158 **(D)** \$1219 **(E)** \$1449

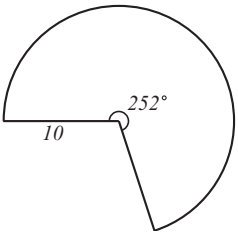
15. A street has parallel curbs 40 feet apart. A crosswalk bounded by two parallel stripes crosses the street at an angle. The length of the curb between the stripes is 15 feet and each stripe is 50 feet long. Find the distance, in feet, between the stripes.

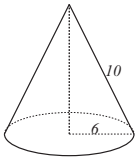
(A) 9 **(B)** 10 **(C)** 12 **(D)** 15 **(E)** 25

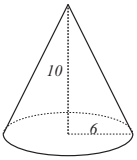
16. The mean of three numbers is 10 more than the least of the numbers and less than the greatest. The median of the three numbers is 5. What is their sum?

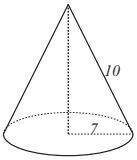
(A) 5 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 36

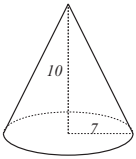
17. Which of the cones below can be formed from a 252° sector of a circle of radius 10 by aligning the two straight sides?

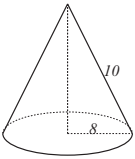


(A) 

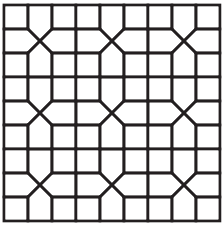
(B) 

(C) 

(D) 

(E) 

18. The plane is tiled by congruent squares and congruent pentagons as indicated. The percent of the plane that is enclosed by the pentagons is closest to



(A) 50 **(B)** 52 **(C)** 54 **(D)** 56 **(E)** 58

19. Pat wants to buy four donuts from an ample supply of three types of donuts: glazed, chocolate, and powdered. How many different selections are possible?

(A) 6 **(B)** 9 **(C)** 12 **(D)** 15 **(E)** 18

20. A regular octagon is formed by cutting an isosceles right triangle from each of the corners of a square with sides of length 2000. What is the length of each side of the octagon?

(A) $\frac{1}{3}(2000)$ **(B)** $2000(\sqrt{2}-1)$ **(C)** $2000(2-\sqrt{2})$

(D) 1000 **(E)** $1000\sqrt{2}$

21. A right circular cylinder with its diameter equal to its height is inscribed in a right circular cone. The cone has diameter 10 and altitude 12, and the axes of the cylinder and cone coincide. Find the radius of the cylinder.

(A) $\frac{8}{3}$ **(B)** $\frac{30}{11}$ **(C)** 3 **(D)** $\frac{25}{8}$ **(E)** $\frac{7}{2}$

22. In the magic square shown, the sums of the numbers in each row, column, and diagonal are the same. Five of these numbers are represented by v, w, x, y, and z. Find $y+z$.

v	24	w
18	x	y
25	z	21

(A) 43 **(B)** 44 **(C)** 45 **(D)** 46 **(E)** 47

23. A box contains exactly five chips, three red and two white. Chips are randomly removed one at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

(A) $\frac{3}{10}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{7}{10}$

24. In trapezoid $ABCD$, $\overline{AB}$ and $\overline{CD}$ are perpendicular to $\overline{AD}$, with $AB+CD=BC$, $AB<CD$, and $AD=7$. What is $AB\cdot CD$?

(A) 12 **(B)** 12.25 **(C)** 12.5 **(D)** 12.75 **(E)** 13

25. How many positive integers not exceeding 2001 are multiples of 3 or 4 but not 5?

(A) 768 **(B)** 801 **(C)** 934 **(D)** 1067 **(E)** 1167

WRITE TO US!

Correspondence about the problems and solutions for this ***AMC 10*** *should be addressed to:*

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Phone: 402-472-2257; Fax: 402-472-6087; email: titu@amc.unl.edu; www.unl.edu/amc

2001 AIME

The AIME will be held on Tuesday, March 27, 2001 with the alternate on April 10,2001. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score in the top 1% of this AMC 10 or receive a score of 100 or above on the AMC 12. Alternately, you must be in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) on Tuesday, May 1, 2001. The best way to prepare for the AIME and USAMO is to study previous years of these exams. Copies may be ordered as indicated below.

PUBLICATIONS

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Examinations: Each price is for one copy of an exam and its solutions for one year. Specify the years you want and how many copies of each. All prices effective to September 1, 2001.

- AMC 10 2000-2001/AHSME (AMC 12) 1989-2001, $1 per copy per year.
- AIME 1989-2001, $2 per copy per year.
- USA and International Math Olympiads, 1989-2000, $5 per copy per year.
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2001

AMC 10

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 13, 2001

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 13.** Nothing is needed from inside this package until February 13.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form A found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC Director, Titu Andreescu, no later than 24 hours following the examination.
4. Please Note: All Problems and Solutions are copyrighted; it is illegal to make copies or transmit them on the internet or world wide web without permission.
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