General Instructions

A. Below are the 8 math areas that appear most often in competitions like Math Kangaroo, AMC 8, Bulgarian Contest, and others:

1. Arithmetic (AR)
2. Algebra (AL)
3. Number Theory (NT)
4. Combinatorics:
    - General Combinatorics (GC) and
    - Combinatorial Geometry (CG)
5. Geometry:
    - Plane Geometry (PG)
    - Spatial Geometry (SG)
6. Optimization (OP)
7. Logic (LO)
8. Probability and Statistics (PR, ST).

Review/study their descriptions, main tasks, objects, and ideas. See if you can connect the problems that you solve with one or more of these areas.

B. Loosely assigning math areas. Many problems can be solved in more than one way, and hence more than one math area can be connected with them. Even one solution may require ideas from different areas of mathematics. It is possible that people will disagree on the math area for certain problems. Thus, view assigning "math areas" to problems as a fun but slightly "imprecise" part of problem-solving that will give you a general feeling and deeper understanding of each problem, especially in how it relates to other problems in mathematics.

Eight Most Common Math Contest Topics

1. **Arithmetic** (from Greek ἀριθμός, *arithmos,* "number") is the oldest and most elementary branch of mathematics, used for tasks from simple day-to-day counting to advanced science and business calculations. It involves the study of quantity, especially as the result of operations that combine numbers. In common usage, it refers to using the four basic operations of *addition, subtraction, multiplication, and division* on small numbers.

2. **Algebra** (from Arabic *al-jebr,* "reunion of broken parts") comes from the idea that one can perform operations of arithmetic with non-numerical mathematical objects. These objects are *variables* representing either numbers that are not yet known (*unknowns*) or unspecified numbers (*indeterminates* or *parameters*). This allows one to state and prove properties that are true no matter which specific numbers are involved.

3. **Number theory** is devoted primarily to the study of the *integers* and is often called *"The Queen of Mathematics"* because of its foundational place in mathematics. Number theorists study *prime numbers* and objects made out of *integers* (e.g., fractions like 3/5 called *rational numbers*), and they solve *Diophantine equations*: equations whose solutions are only integers. Number theory is often considered to have given birth to *computer science*.

4. **Combinatorics** is the art of counting objects by type, size and other common properties. Combinatorics solves problems from many other areas of mathematics, science, and computer science. In the later 20th century, powerful development made it into an independent branch of mathematics.

5. **Geometry** (Ancient Greek: *geo-* "earth", *-metron* "measurement") is concerned with questions of *shape, size, relative position of figures, and the properties of space*. Geometry arose independently in a number of early cultures as a body of practical knowledge concerning *lengths, areas, and volumes*. By the 3rd century BC geometry was put into an axiomatic form by *Euclid*. *Archimedes* developed ingenious techniques for calculating areas and volumes. The field of *astronomy* poses lots of geometric problems.

6. **Optimization** is the selection of a best element (for example, the largest or smallest element) from a bunch of things. Optimization is the foundation of *economics* and *ecology*, and it applies to any science and area of human endeavor, from day-to-day life activities such as taking the shortest route to school to global questions of most efficiently supplying food to Earth's population, or choosing the most efficient orbit for a satellite.

7. **Logic** (from the Greek λογική, *logike*) describes the use of valid reasoning in some activity and is featured most prominently in *philosophy*, *mathematics*, and *computer science*. Logic was studied in ancient India, China, Persia, and Greece. In the West, logic was established as a formal discipline by *Aristotle*, who gave it a fundamental place in philosophy.

8. **Probability and Statistics** are two related but separate disciplines.

Probability theory studies the likelihood than an event will occur, i.e., it answers rigorously the questions "Will a specific event occur?" and "How certain are we that the event will occur?" The level of our "certainty" is called the *probability* of the event to occur. It is a number between 0 and 1.

$$\textbf{Probability of our event} = \frac{\text{number of desirable outcomes}}{\text{total number of outcomes}}.$$

Statistics is the study of the collection, organization, analysis, interpretation and presentation of data. Its mathematical foundations were laid in the 17th century with the development of probability theory by *Blaise Pascal* and *Pierre de Fermat*. Mathematical probability theory arose from the study of games of chance, although the concept of probability was already examined in *medieval law* and by *philosophers* such as *Juan Caramuel*. Today, statistics is widely employed in *government, business, and natural and social sciences*.

A main task in statistics is to calculate various **averages** for given numbers (or data). The most common average is the so-called

$$\textbf{Arithmetic mean (or just average)} = \frac{\textbf{add all numbers}}{\textbf{how many numbers you have}} = \frac{A_1 + A_2 + A_3 + \ldots + A_n}{n}.$$

Median = the **middle** of all numbers when arranged in increasing order:

- When n *is odd* there will be a unique middle number. For example, if we have 5 numbers total, then the 3rd number will be the middle number and hence it will be the median: arranging $A_1 \leq A_2 \leq A_3 \leq A_4 \leq A_5$ gives
$$\textbf{median} = A_3.$$
- When n *is even* there will be two "middle numbers". For example, if we have 6 numbers total, then the 3rd and the 4th number will compete to be the middle numbers: arranging $A_1 \leq A_2 \leq A_3 \leq A_4 \leq A_5 \leq A_6$ makes A_3 and A_4 the two middle numbers. The median will be the average of A_3 and A_4:
$$\textbf{median} = \frac{A_3 + A_4}{2}.$$

Mode = The value that appears most often. It may or may not be unique. For example, in the set {1,2,2,5,6,7,8,8,8,10}, the mode is 8; but in the set {1,2,2,5,5,5,6,7,8,8,8,10} there are two modes: 5 and 8.