


WARM-UP

Can you cut an arrow-shape out of a piece of paper with a single straight-line cut?



Step 1: Draw an arrow
Step 2: Attempt to fold it up so that you can cut it out all at once.

AWESOME TWIST-AND-CUT RESULTS FROM @HOME EXPERIMENTS?



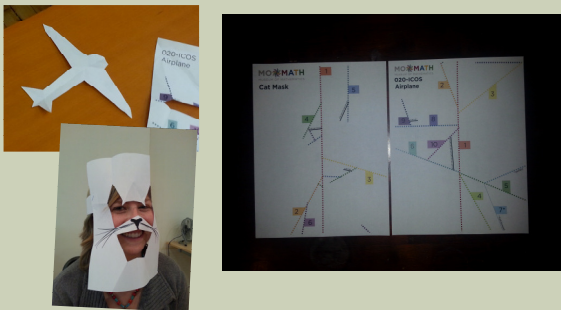
EUCLID AND DEMAINE

Computational
Geometry and
Origami


THE ONE-CUT ARROW

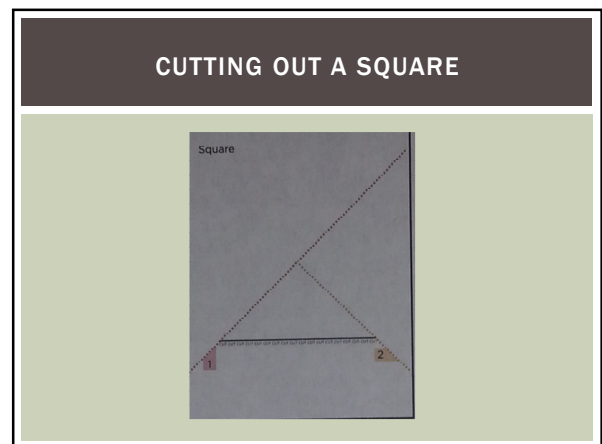
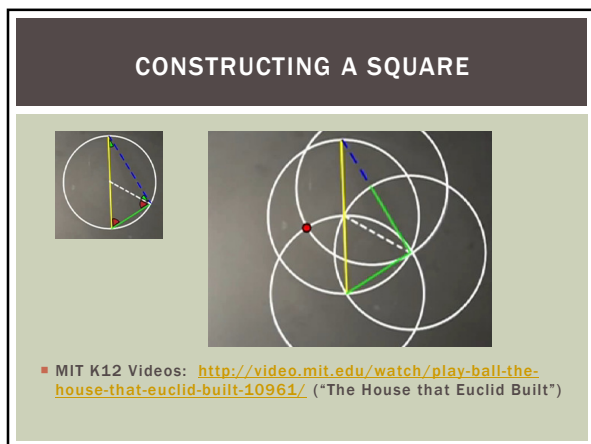
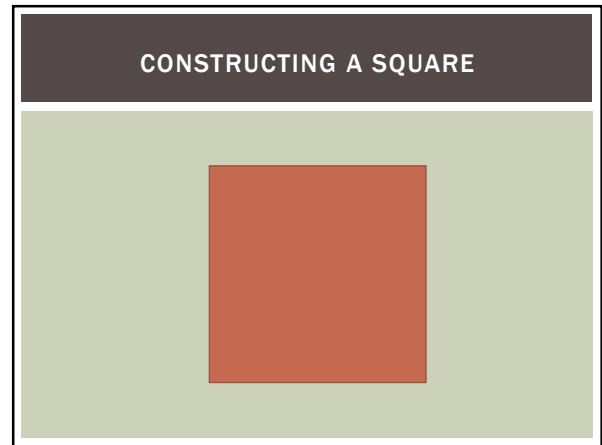
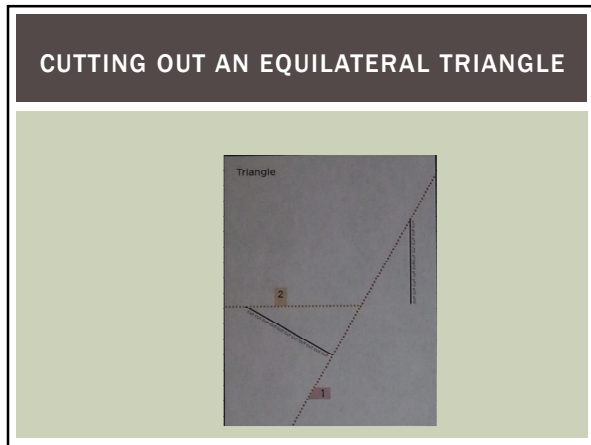
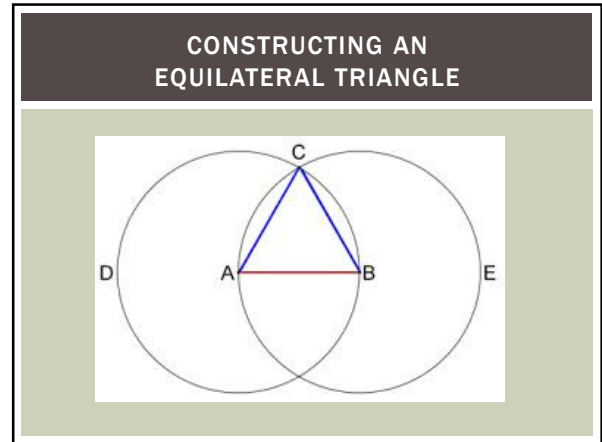
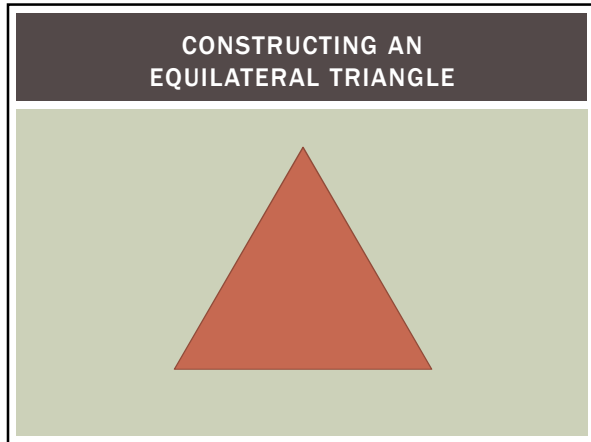


ONE-CUT POLYGONS

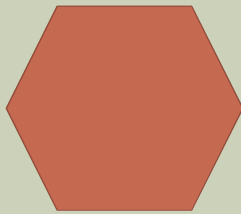


REGULAR, CONVEX POLYGONS

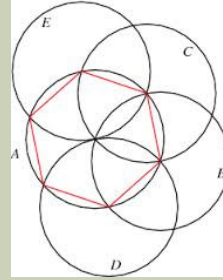




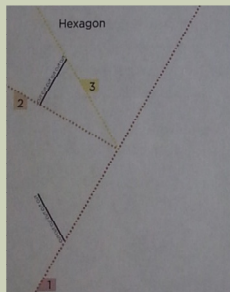
CONSTRUCTING A REGULAR HEXAGON



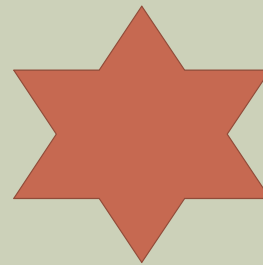
CONSTRUCTING A REGULAR HEXAGON



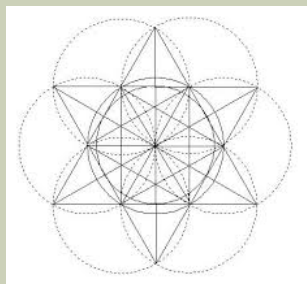
CUTTING OUT REGULAR HEXAGON



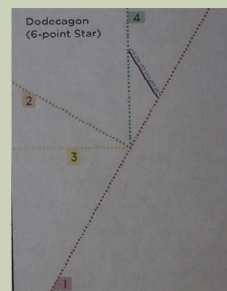
CONSTRUCTING A 6-PT STAR



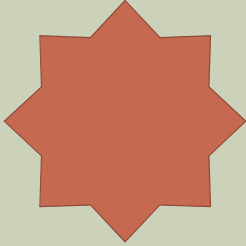
CONSTRUCTING A 6PT STAR



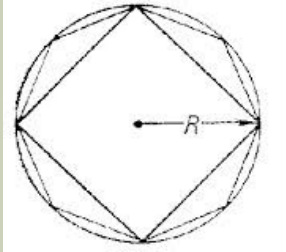
CUTTING OUT A 6PT STAR



CONSTRUCTING A REGULAR OCTAGON OR AN 8-POINTED STAR



CONSTRUCTING A REGULAR OCTAGON OR AN 8-POINTED STAR



CUTTING OUT A REGULAR N-GON

How many folds does it take to cut out a regular N-GON - what angles do you make the folds and the cut at?

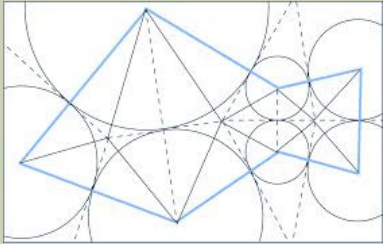
Extra Extension: How many different-looking crease patterns that use the minimal number of folds make the same regular polygons?

PART 2: IRREGULAR POLYGONS

And "flat foldability"

IT'S ALL ABOUT...

CIRCLES!



ONE-CUT EXAMPLES THAT REQUIRE PRE-CREASING

MOUNTAIN VS. VALLEY

Mountain fold.

Valley fold.

TRY ONE OUT

1D FLAT FOLDING – COLLEGIATE LEVEL COMPUTATIONAL ORIGAMI

Folding a strip of paper – what patterns and spacing of mountain and valley folds are possible?

Step 1: Find a crease pattern that's impossible to fold

Super Extra Spicy:
Step 2: Characterize ALL of the patterns that cannot be folded

Super Extra Spicy:
What crease patterns can be flat folded if the paper is a ring instead of a finite segment?

2D FLAT FOLDING

Single-vertex crease pattern (without loss of generality)
= disk of paper,
creases emanate from center

Idea: capture local foldability around a vertex
= circular sequence of angles $\theta_1, \theta_2, \dots, \theta_n$
- normally, $\theta_1 + \theta_2 + \dots + \theta_n = 360^\circ$
- allow other sums, especially $\leq 360^\circ$ (convex cone)
in particular for induction

Eric Demaine's Lectures are available online, this is from the 3rd lecture:
courses.csail.mit.edu/6.849/fall10/lectures/L03.pdf

2D FLAT FOLDING

Single Vertex Crease Patterns:

General Theorem: A single vertex crease pattern is foldable by angles $\theta_1, \theta_2, \theta_3, \dots, \theta_n$ is flat foldable if and only if n is even and the sum of the odd angles is equal to the sum of the even angles.

Mountain/Valley Assignments:
M-V = +/- 2

2D FLAT FOLDING

WORKING IT OUT IN STAGES

- 1) Why must the degree be even?
- 2) A Special Case: A 4-degree vertex: prove that opposite segments must each be a pair of angles that sum to 180.
- 3) Generalize to 2n-degree vertices
- 4) Why does $M-V = +/- 2$

6.849 FIRST PROBLEM SET, FIRST PROBLEM

Problem 1. Which of the following crease patterns are flat foldable? Are any simply foldable (foldable by a sequence of simple folds)? Justify each answer by either submitting a flat folding or arguing why the crease pattern cannot fold flat.

(a) <http://www.math.ucdavis.edu/~math/preceptor04/>

(b) <http://www.math.ucdavis.edu/~math/preceptor04/>

(c) <http://www.math.ucdavis.edu/~math/preceptor04/>

(d) <http://www.math.ucdavis.edu/~math/preceptor04/>

FOLDING A FRACTAL

Instructions:

- Take a strip of paper
- Fold it in half to the right
- Fold the result in half to the right
- ...etc...
- Unfold, but don't flatten the strip – correct each crease until it is exactly a right angle.

Exercise:
What will the pattern of Mountain vs. Valley creases be?

FOLDING A FRACTAL

0 1 2 3 4 5 6 7

FOLDING A FRACTAL

The Dragon Curve:

MORE FRACTAL FOLDING AND TILING

MORE FRACTALS: THE HILBERT CURVES

Vi Hart
 "Squiggle Inception"
<https://www.youtube.com/watch?v=ik2CZosAw28>

MORE FRACTALS: THE APOLLONIAN GASKET

IN THE NEXT ROUND OF SESSIONS... TESSELLATIONS!

TESSELLATIONS

Tessellation: a set of shapes which fit together to cover a surface without any gaps or overlaps.

Monohedral Tessellation: a tessellation in which all of the tiles are congruent (exactly the same)

PARQUET DEFORMATIONS

Arabic Mosaics

Temari Balls

Escher Drawings