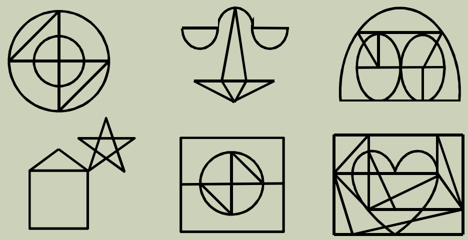
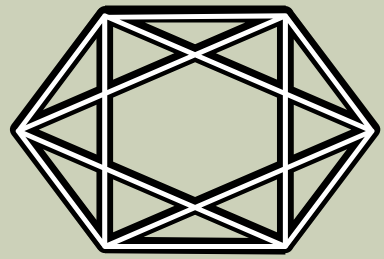
1ST CLASS STARTER – 9/30/14

Laser-cutting: for which of these pictures is it possible to trace the entire shape with a single, non-back-tracking line?



QUICK EXAMPLE

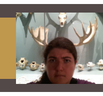


GRAPHS I

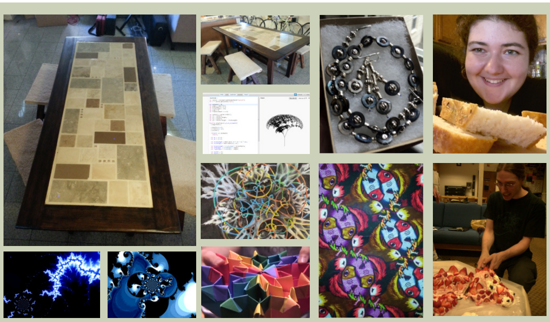
The Eulerian and The Hamiltonian

This presentation can be found @ https://sites.google.com/site/applelessgarden/mathcircle

HI! I'M ZANDRA VINEGAR



I LIKE TO MAKE THINGS



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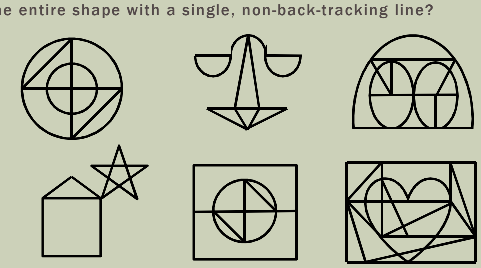
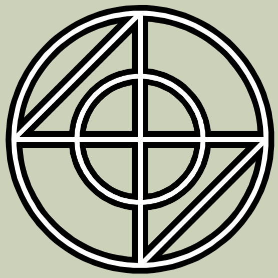
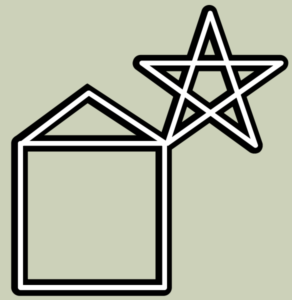


SOLUTION

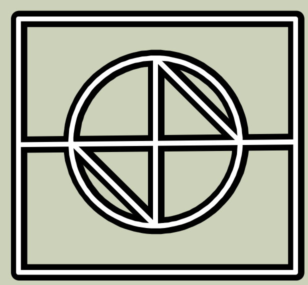


SOLUTION

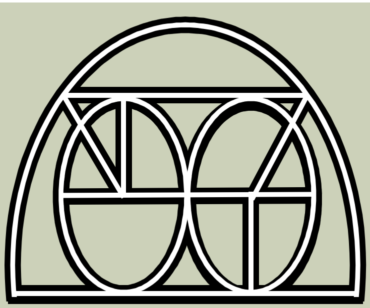


SOLUTION

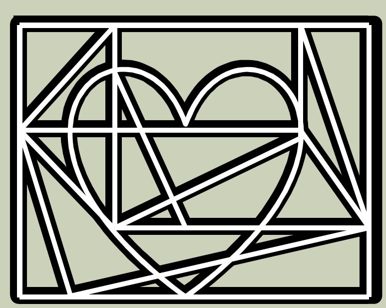
SOLUTION



SOLUTION



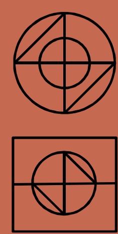
SOLUTION



"ASKING 'WHAT IF…'"

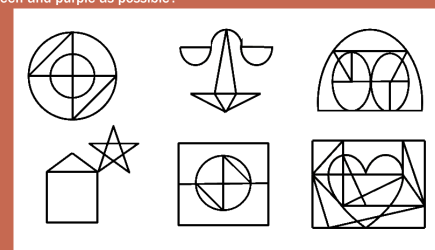
What questions do you now have about this mathematical pattern?

- Try doing something else with the same graphs…
- Try making your own graphs in a way that tests or modifies the known pattern…
- Try tweaking the rules just a little…



COLORING

Stained-glass: blue glass is free, orange is very cheap, and green is very expensive, and purple is very, very expensive. How can you color this whole panel to use as little green and purple as possible?



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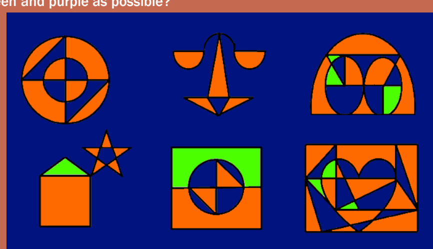
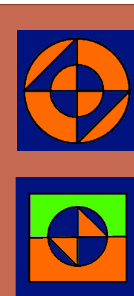


DIGGING DEEPER

What questions do you now have about this mathematical pattern?

- Can you make a stained glass pattern that requires more than 3 colors? How many colors might you need
- Can you make a stained glass pattern for which the degree of every intersection is even, that requires 3 or more colors?

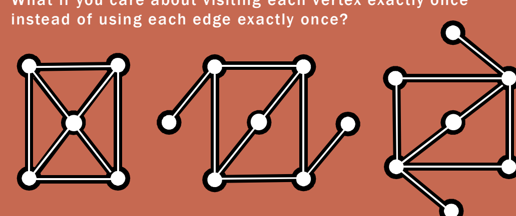


VOCABULARY

- Graph
  - Vertex
  - Edge
  - Region
- Degree of a Vertex
- Eulerian Path
- Eulerian Cycle
- Chromatic Number
- Four Color Theorem

DIGGING DEEPER

What if you care about visiting each vertex exactly once instead of using each edge exactly once?

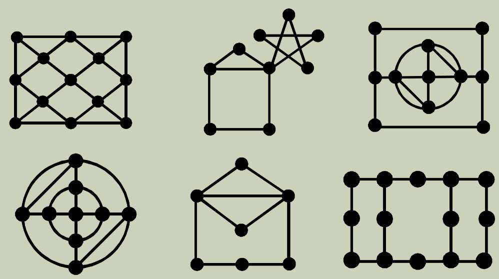


This is called a "Hamiltonian Path"

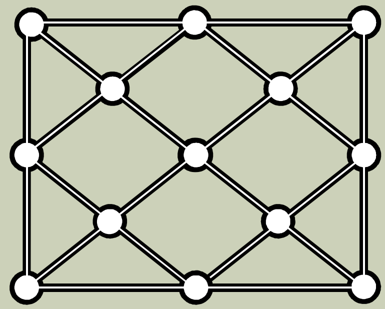
If the journey ends where it begins, then it's called a "Hamiltonian Cycle"

HAMILTONIAN PATH OR CYCLES

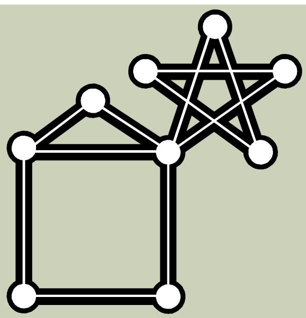
Which of these graphs has a Hamiltonian path and/or cycle?



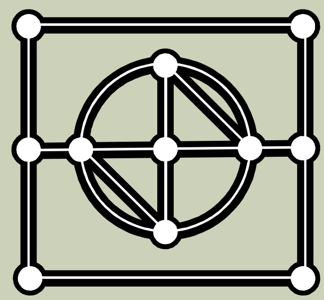
SOLUTION



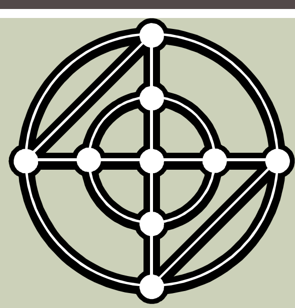
SOLUTION



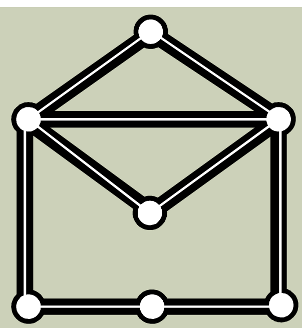
SOLUTION



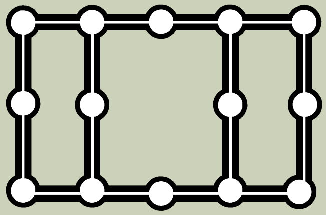
SOLUTION



SOLUTION



SOLUTION



HAMILTONIAN VS. EULERIAN

	Hamiltonian Path	Hamiltonian Cycle	Eulerian Path	Eulerian Cycle
A	–X–	–X–	–X–	–X–
B	–X–	–X–	–X–	no
C	–X–	no	–X–	–X–
D	–X–	no	–X–	no
E	–X–	–X–	no	no
F	no	no	–X–	–X–
G	–X–	no	–X–	no
H	–X–	no	no	no
I	no	no	–X–	no
J	no	no	no	no

ADDING EDGES

How many edges (at most) can you need to add in order to make a graph that does not have a Hamiltonian path into one that does?

- When do you need to add many additional edges?
- When do you only need to add one or a few?

With 5 vertices, how many edges can you add before there *must* be a Hamiltonian path?

ORE'S THEOREM

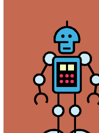
In a graph with $n \geq 3$ vertices, if for each pair of vertices either $\deg(u)+\deg(v) \geq n$ or u and v are adjacent, then the graph has a Hamilton circuit.

COROLLARY: DIRAC'S THEOREM

In a graph with $n \geq 3$ vertices, if each vertex has $\deg(v) \geq n/2$, then the graph has a Hamilton circuit.

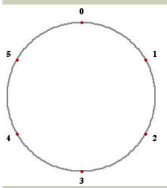
SAME LENGTH LINES

What if every time you move, you have to move the same amount – like a robot that can only make one size of step?

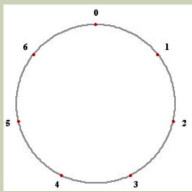


EULERIAN STARS

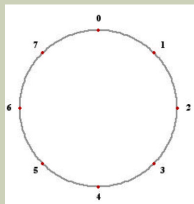
POINTS: 6
JUMP: 2



POINTS: 7
JUMP: 2



POINTS: 8
JUMP: 2



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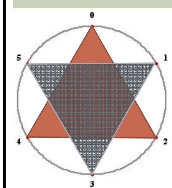
JUMP: 2



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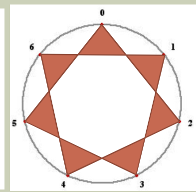
JUMP: 2



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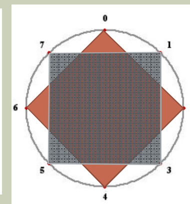
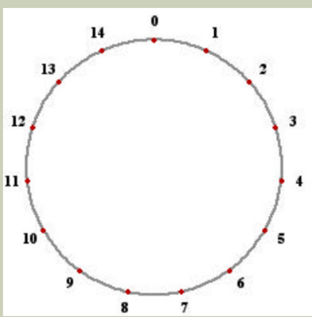
JUMP: 2



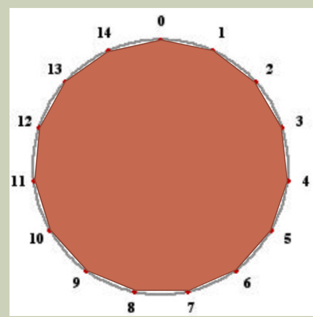
EULERIAN STARS

POINTS: 15
JUMP: ___?



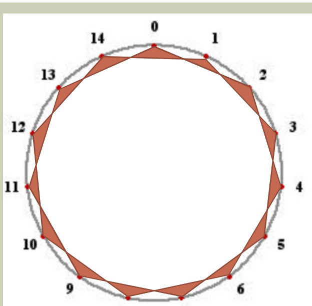
EULERIAN STARS

POINTS: 15
JUMP: 1



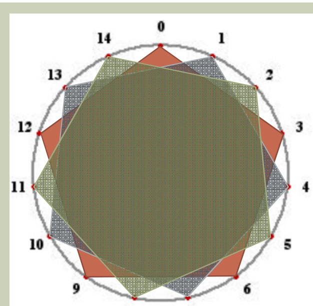
EULERIAN STARS

POINTS: 15
JUMP: 2



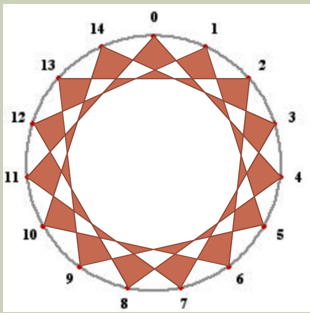
EULERIAN STARS

POINTS: 15
JUMP: 3



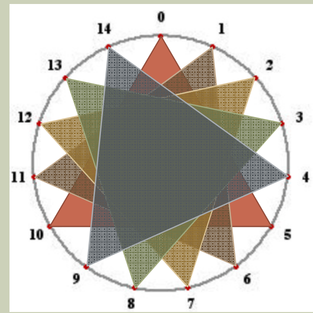
EULERIAN STARS

POINTS: 15
JUMP: 4



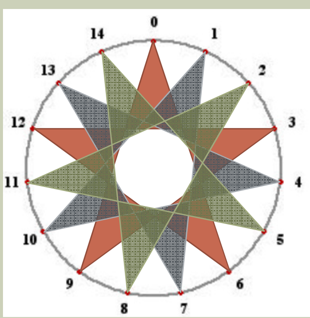
EULERIAN STARS

POINTS: 15
JUMP: 5



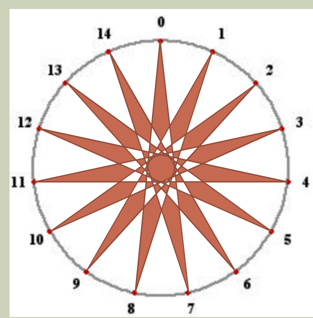
EULERIAN STARS

POINTS: 15
JUMP: 6



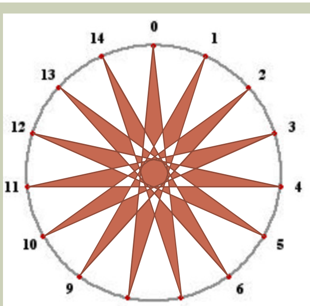
EULERIAN STARS

POINTS: 15
JUMP: 7



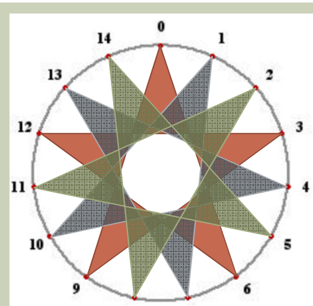
EULERIAN STARS

POINTS: 15
JUMP: 8



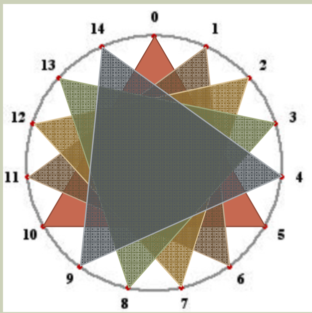
EULERIAN STARS

POINTS: 15
JUMP: 9



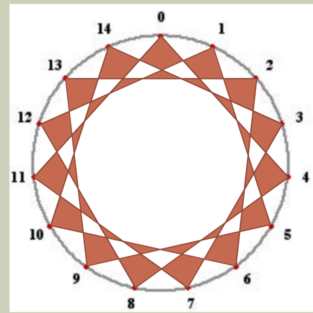
EULERIAN STARS

POINTS: 15
JUMP: 10



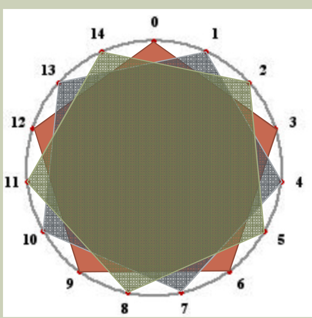
EULERIAN STARS

POINTS: 15
JUMP: 11



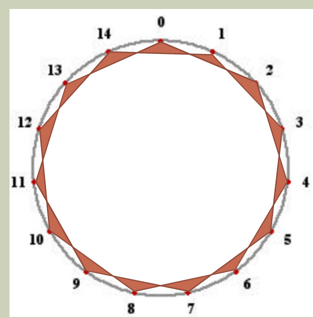
EULERIAN STARS

POINTS: 15
JUMP: 12



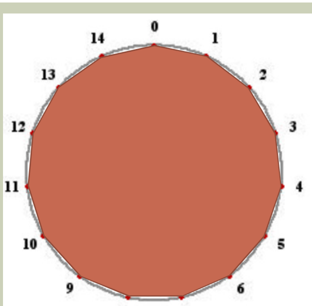
EULERIAN STARS

POINTS: 15
JUMP: 13



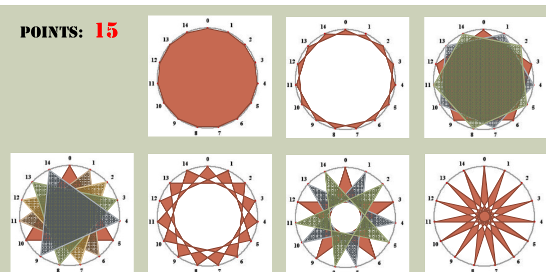
EULERIAN STARS

POINTS: 15
JUMP: 14



EULERIAN STARS

POINTS: 15



EULERIAN STAR COUNTING

- How many *different* 40-point Eulerian stars can be drawn?
- How many *different* 300-point Eulerian stars can be drawn?