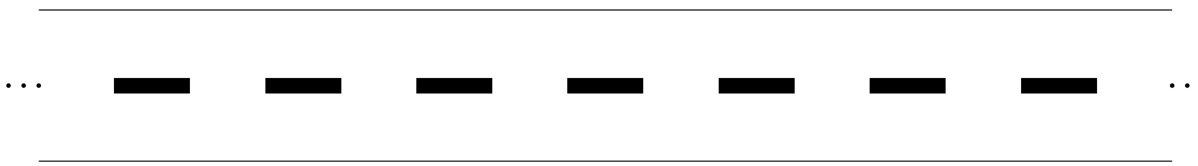
SYMMETRY GROUPS (PART 3)

FRIEZE PATTERNS

Consider an infinitely long strip that has a repeated pattern. For example:



If you **translate** this strip to the right, you get the strip back. We say the strip has **translational symmetry**.

If you **reflect** this strip about a line drawn horizontally through the middle, you get the strip back. The strip has **horizontal reflection symmetry**.

If you **reflect** this strip about a line vertically through the middle of one of the black bars, you get the strip back. The strip has **vertical reflection symmetry**.

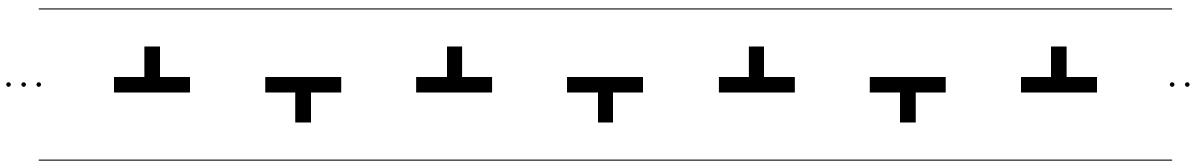
If you **rotate** this strip 180 degrees, you get the strip back. The strip has **rotational symmetry**.

Exercise 1. Draw a strip which has translational symmetry, but none of the other symmetries mentioned above.

Exercise 2. Draw a strip which has only translation symmetry and horizontal reflection symmetry.

Exercise 3. What symmetries does the following infinite strip have:

There is a fifth kind of symmetry called a **glide reflection symmetry**, which is a combination of a translation and a horizontal reflection. Here is an example of a strip with glide reflection symmetry:



Exercise 4. What other symmetries does that strip have?

The translation which is a part of the glide reflection symmetry is not itself allowed to be a symmetry of the strip. So the original strip, for example, does not have glide reflection symmetry.

Exercise 5. Draw a strip that has only translational symmetry and glide reflection symmetry.

Exercise 6. What type of symmetry results when you combine a vertical reflection and a horizontal reflection? Is it possible to draw a strip that has only translational symmetry, vertical reflection symmetry, and horizontal reflection symmetry?

Exercise 7. What type of symmetry results when you combine a horizontal reflection and a rotation?

Exercise 8. What type of symmetry results when you combine two different vertical reflections?

Exercise 9. Draw as many different strips as you can which have different collections of symmetries. How many possibilities can you find? Can you prove that some collections are impossible to obtain?

Now we will investigate the symmetry group of a cube! To investigate this group we will use dice. In order to write down a symmetry, you need to be able to write down a position of a die.

There are six sides: Front (F), Back (B), Left (L), Right (R), Top (T), Bottom (Bo). So, for example you could write down a position of the die:

F: 1
B: 6
L: 2
R: 5
T: 3
Bo: 4

And you could write a symmetry:

$$\begin{array}{lcl} F:1 & & F:2 \\ B:6 & & B:5 \\ L:2 & \longrightarrow & L:6 \\ R:5 & RT & R:1 \\ T:3 & & T:3 \\ Bo:4 & & Bo:4 \end{array}$$

RT stands for "Rotate Top", which means rotate 90 degrees counterclockwise about the top face.

Question 10. Experiment with the die. Write down as many symmetries as you can. Rotate around other faces. Try rotating around edges or vertices. Give a name to each symmetry you find.

Question 11. Find a pair of symmetries that do not commute. That means, two symmetries A and B such that $A + B$ and $B + A$ are different symmetries.

Challenge. How many symmetries of the cube are there?

Question 12. There are three symmetries of the cube which involve rotating the cube 180 degrees around a pair of opposite faces. Give each of these symmetries names. Show that if you combine any two of these symmetries, you get one of these symmetries, or the 'Do Nothing' symmetry. We say that these symmetries are a **subgroup** of the symmetry group of the cube.

Question 13. There is a corner of the die on which the numbers 1, 2 and 3 meet. Write a symmetry of the die that keeps that corner in the same place.

Question 14. For the symmetry in question 4, can you figure out a way of writing it as a sum of rotations around faces?