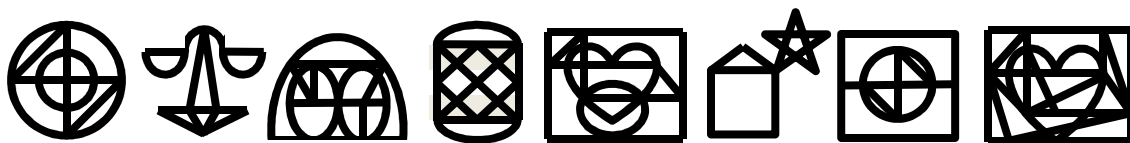


**Laser-cutting:** for which of these pictures is it possible to trace the entire shape with a single, non-back-tracking line?



# Vocabulary

Graph

Degree of a Vertex

Eulerian Path

Eulerian Cycle

Hamiltonian Path

Hamiltonian Cycle

Planar Graph

Dual Graph

Euler's Formula

## The Eulerian and the Hamiltonian

Graphs I  
2014

**Zandra Vinegar**  
[ch3cooh@alum.mit.edu](mailto:ch3cooh@alum.mit.edu)

**Find the full presentation at:**  
<https://sites.google.com/site/applelessgarden/mathcircle>

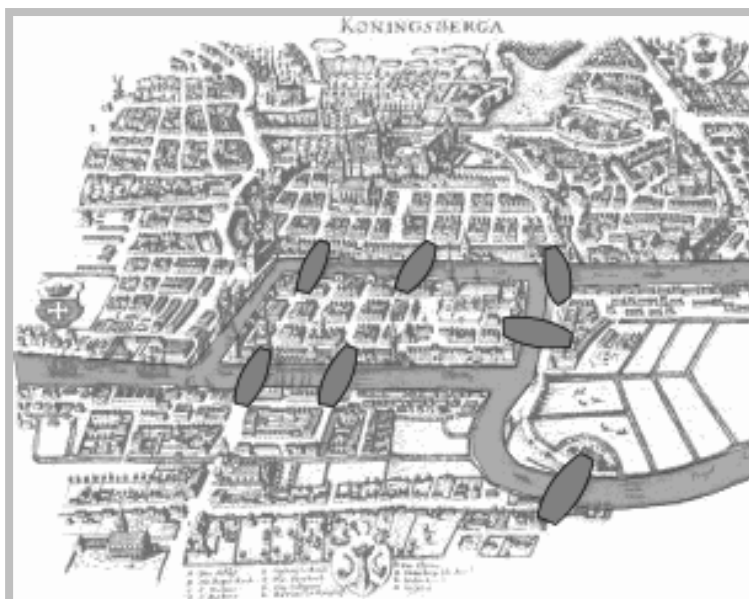
## Extra Challenges

1. Construct a graph with each of the following properties or if such a graph cannot be constructed, explain why:

|   | Hamiltonian Path | Hamiltonian Cycle | Eulerian Path | Eulerian Cycle |
|---|------------------|-------------------|---------------|----------------|
| A | --X--            | --X--             | --X--         | --X--          |
| B | --X--            | --X--             | --X--         | no             |
| C | --X--            | no                | --X--         | --X--          |
| D | --X--            | no                | --X--         | no             |
| E | --X--            | --X--             | no            | no             |
| F | no               | no                | --X--         | --X--          |
| G | --X--            | no                | --X--         | no             |
| H | --X--            | no                | no            | no             |
| I | no               | no                | --X--         | no             |
| J | no               | no                | no            | no             |

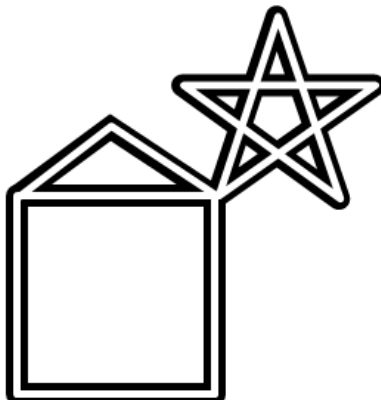
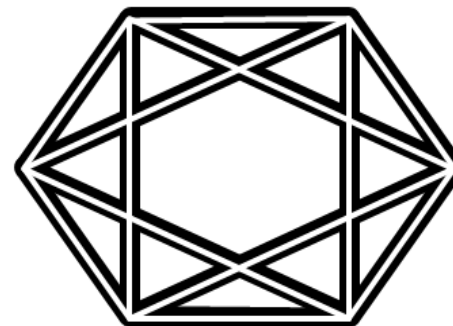
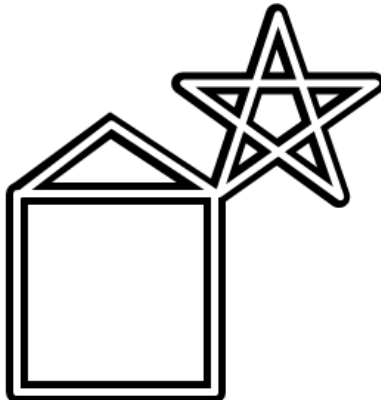
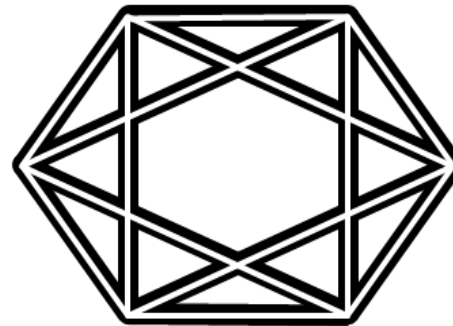
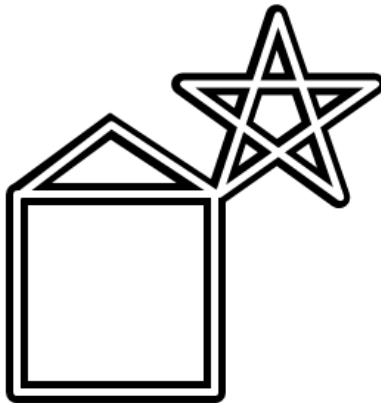
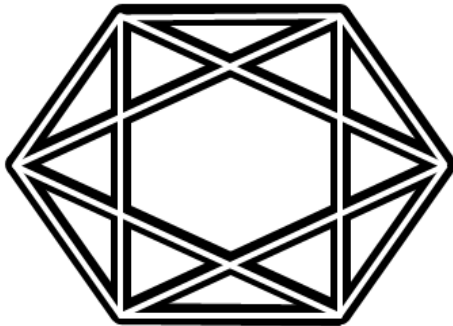
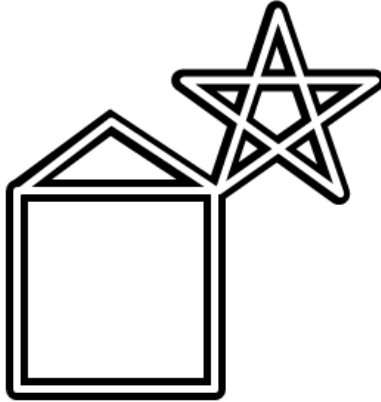
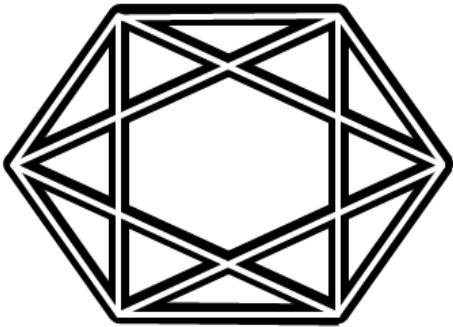
2. Construct a graph on 8 vertices with *as many edges as possible*, but such that the graph does not have a Hamiltonian path.

3. Is it possible to tour the city of Königsberg crossing over each of Königsberg's 7 bridges exactly once? If so, how; if not, why not?



# Finding Eulerian Paths

**Laser-cutting:** for which of these pictures is it possible to trace the entire shape with a single, non-back-tracking line?



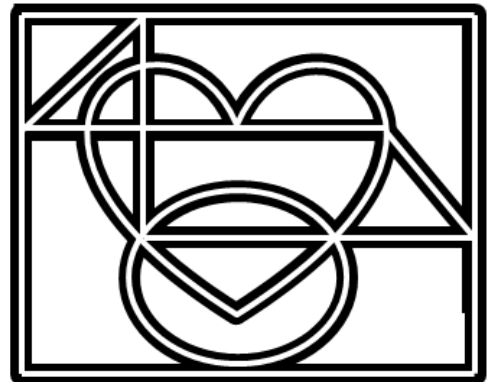
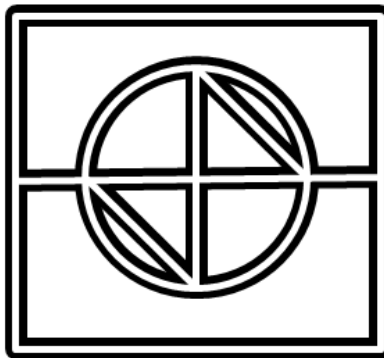
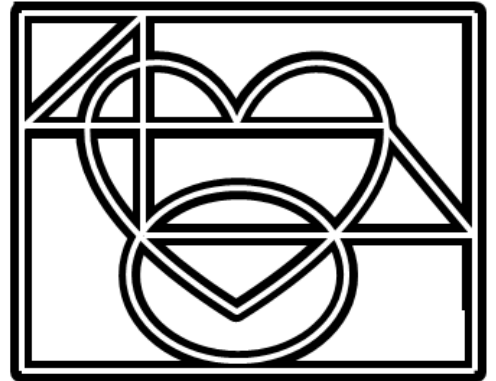
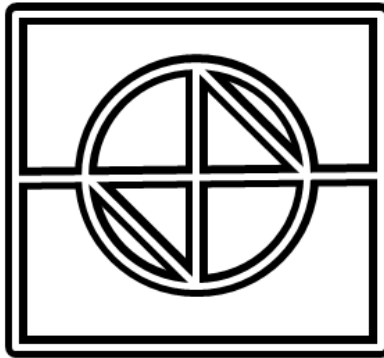
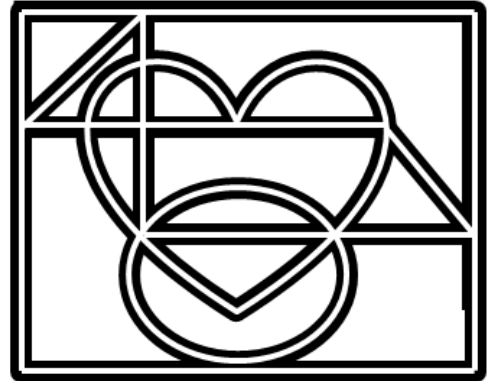
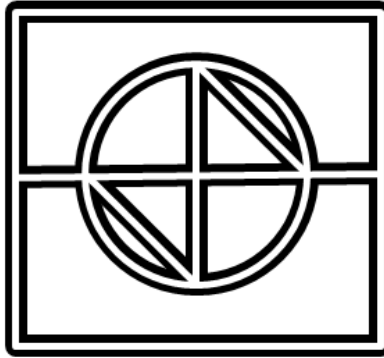
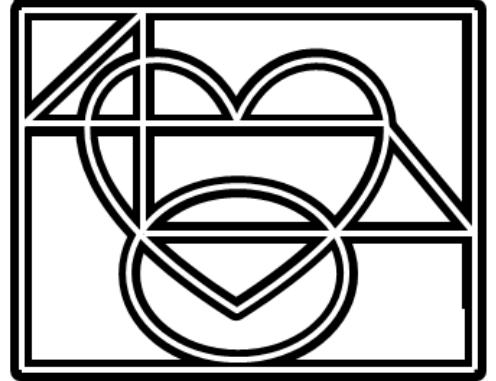
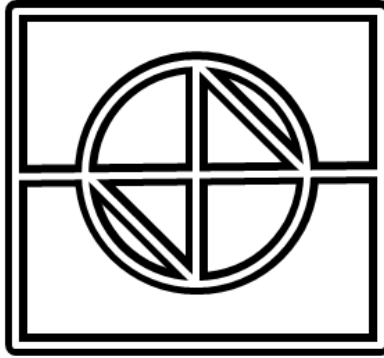
*Path / Cycle / Neither*

*Path / Cycle / Neither*

*Path / Cycle / Neither*

# Finding Eulerian Paths

**Laser-cutting:** for which of these pictures is it possible to trace the entire shape with a single, non-back-tracking line?



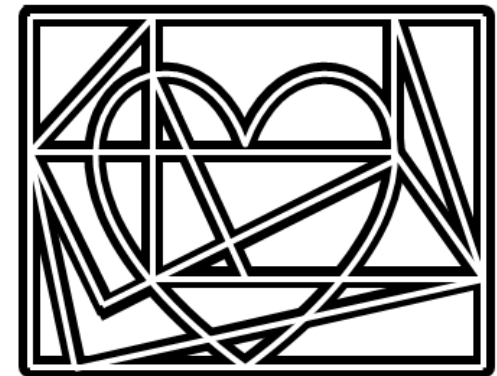
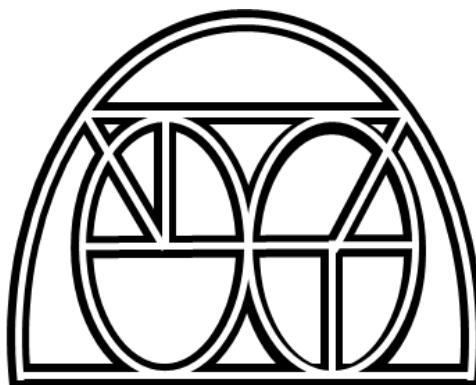
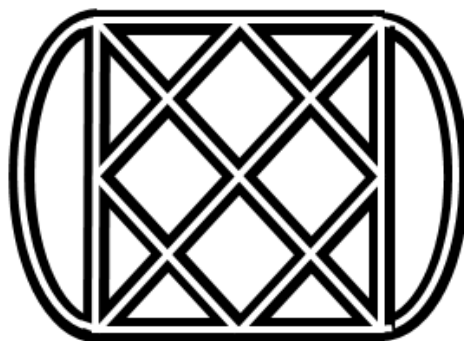
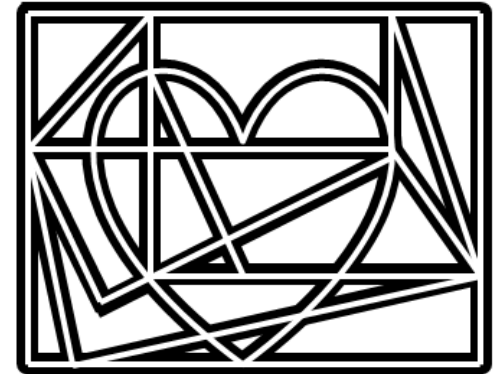
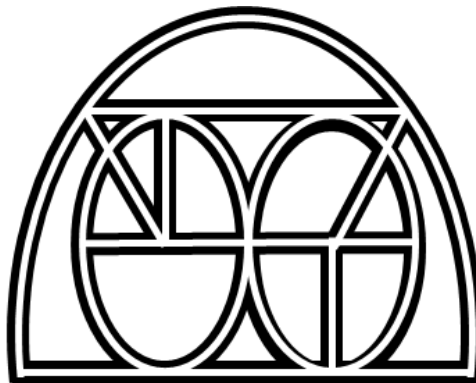
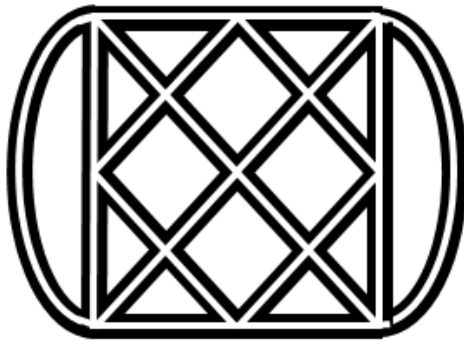
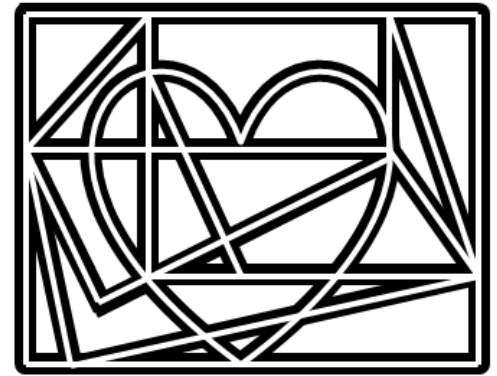
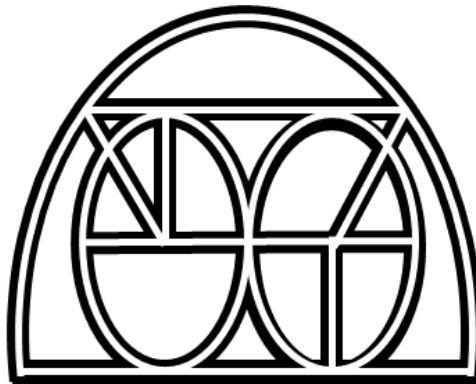
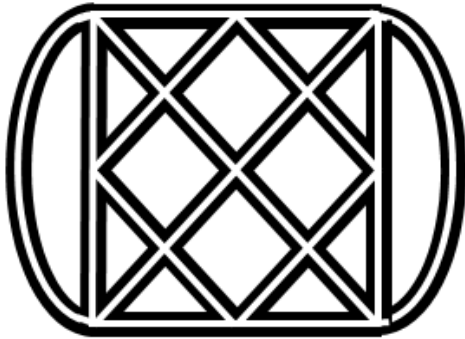
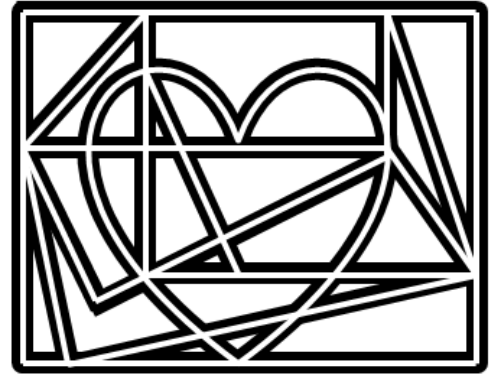
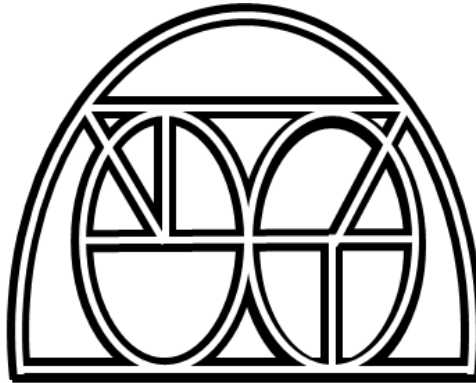
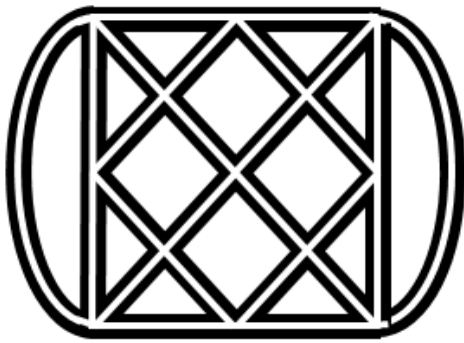
*Path / Cycle / Neither*

*Path / Cycle / Neither*

*Path / Cycle / Neither*

# Finding Eulerian Paths

**Laser-cutting:** for which of these pictures is it possible to trace the entire shape with a single, non-back-tracking line?

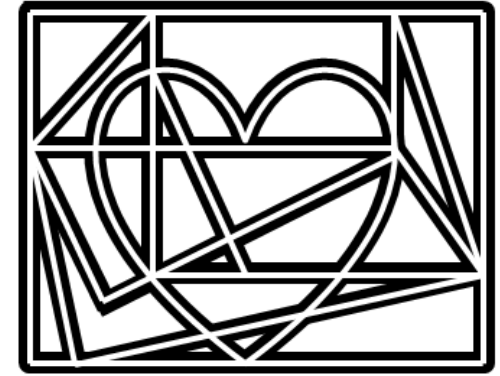
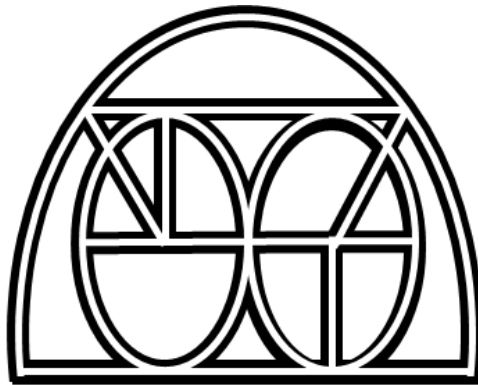
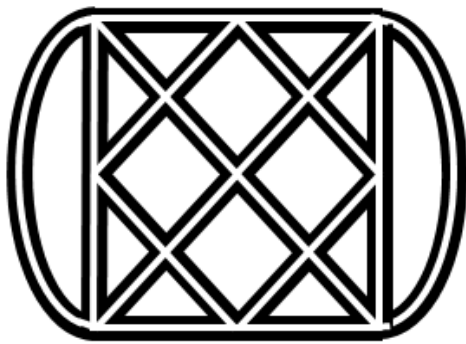
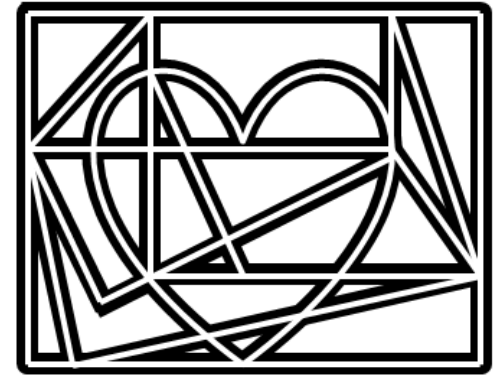
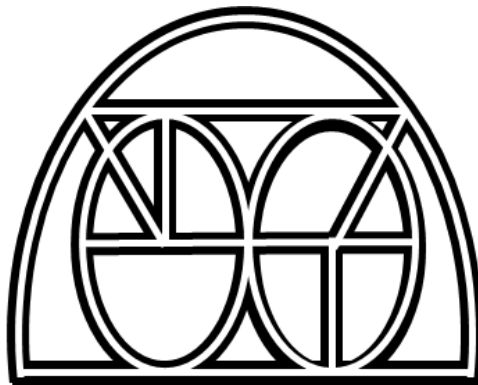
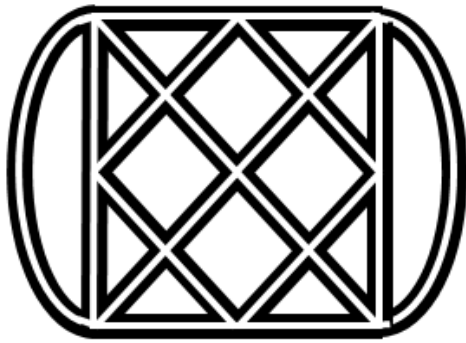
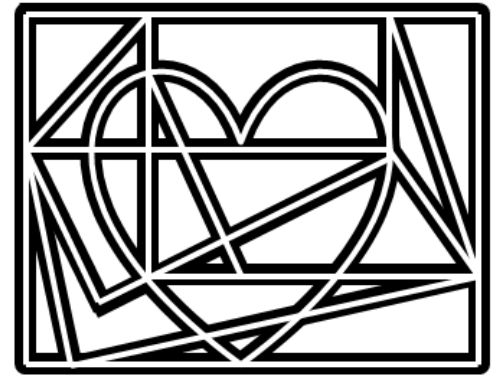
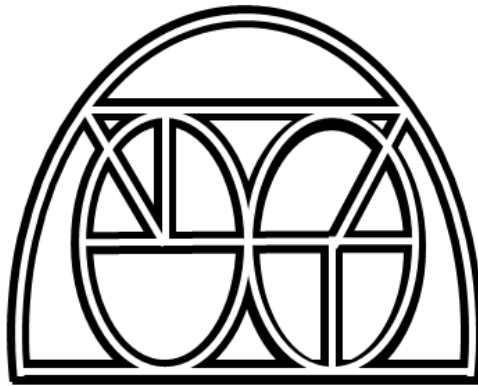
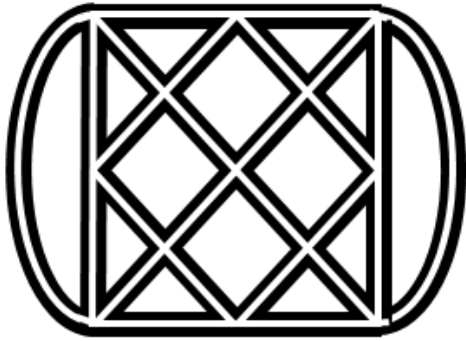
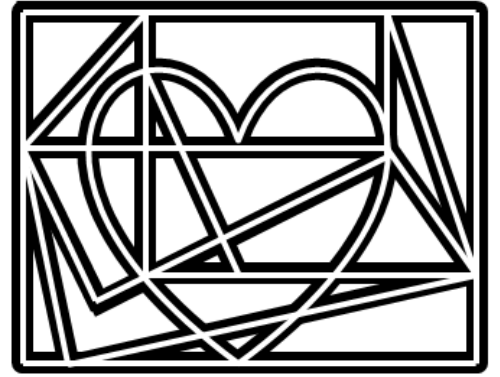
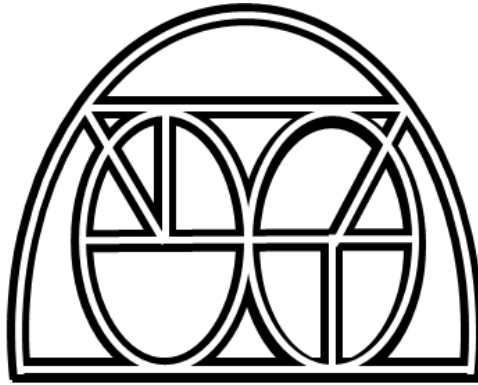
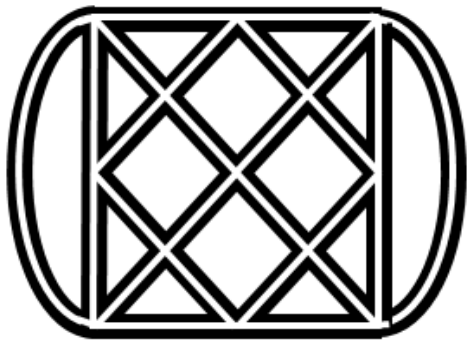


*Path / Cycle / Neither*

*Path / Cycle / Neither*

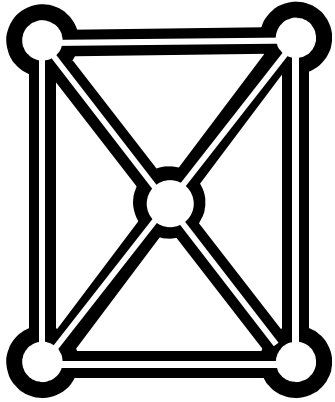
*Path / Cycle / Neither*

Additional Copies:

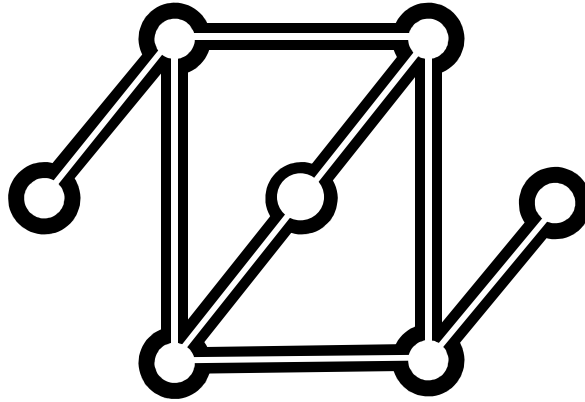


# Finding Hamiltonian Paths

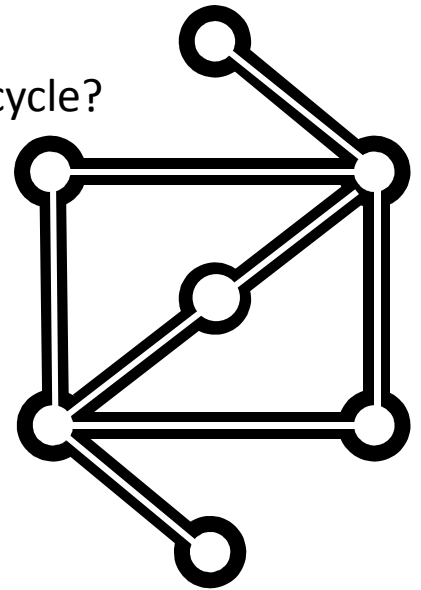
Which of these graphs has a Hamiltonian path and/or cycle?



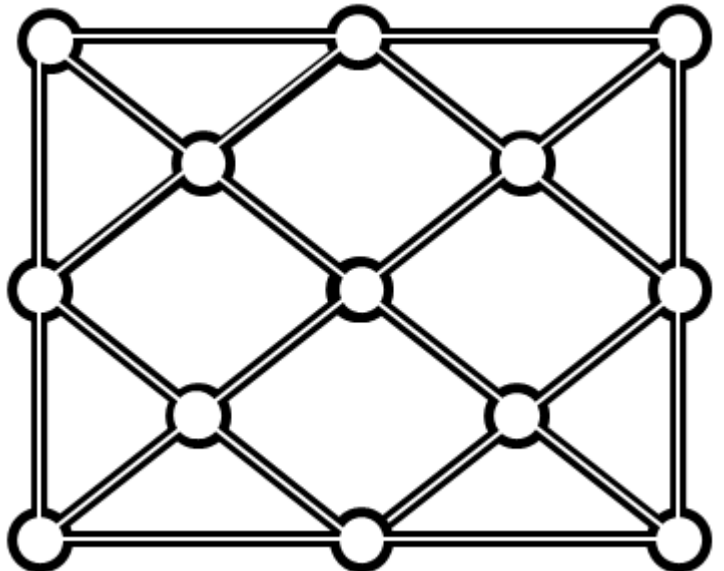
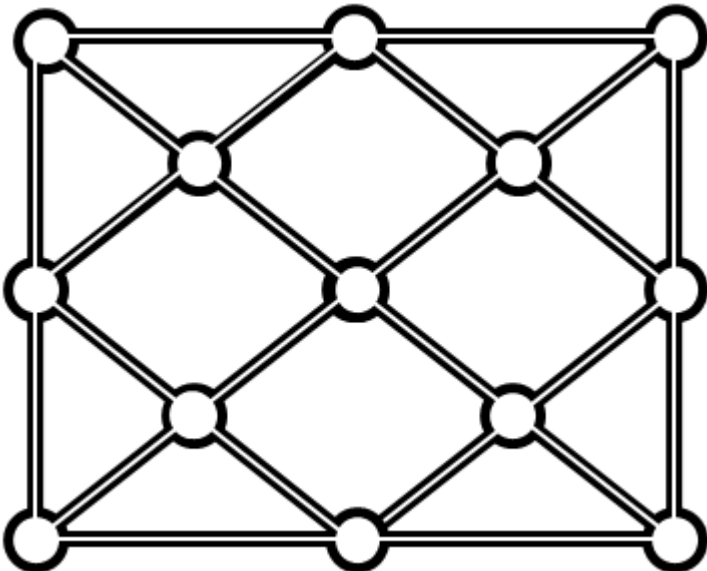
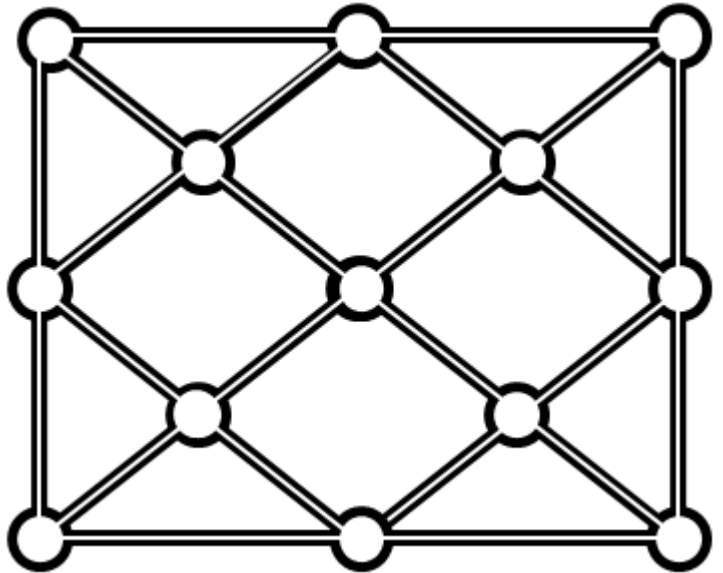
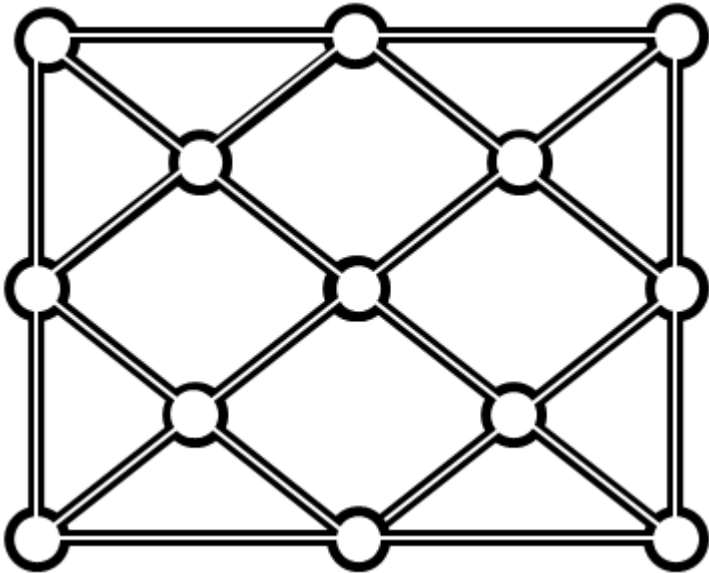
*Path / Cycle / Neither*



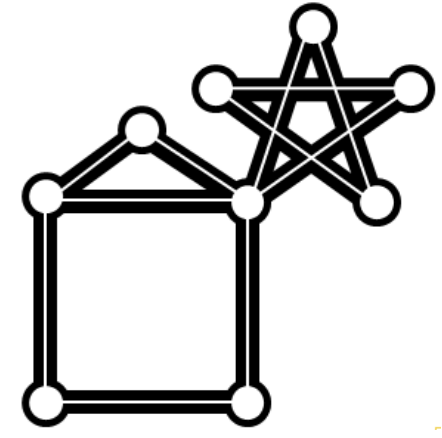
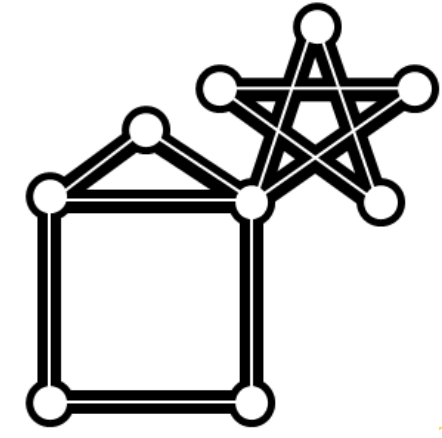
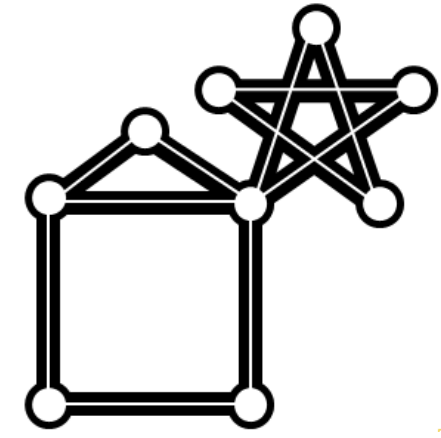
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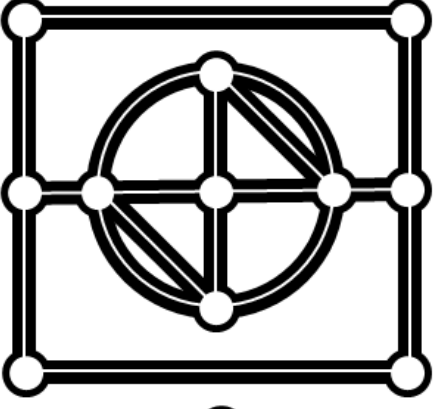
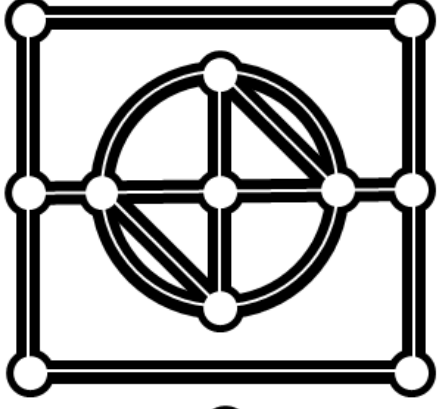
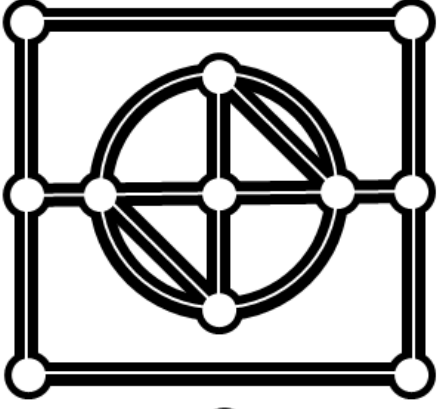
*Path / Cycle / Neither*



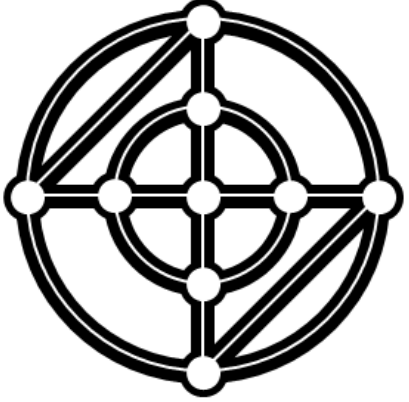
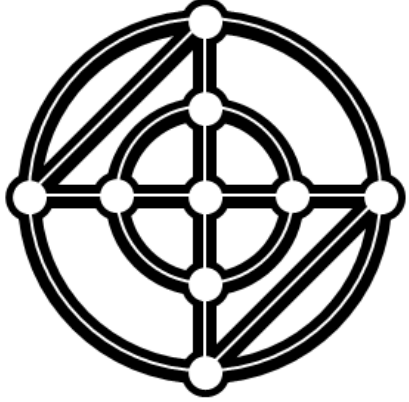
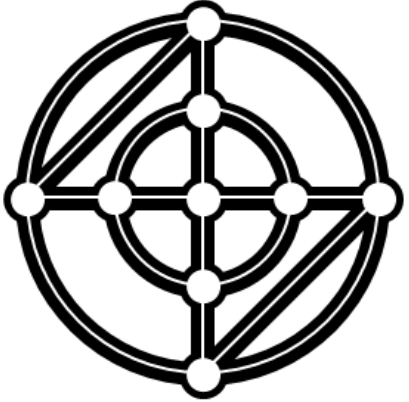
*Path / Cycle / Neither*



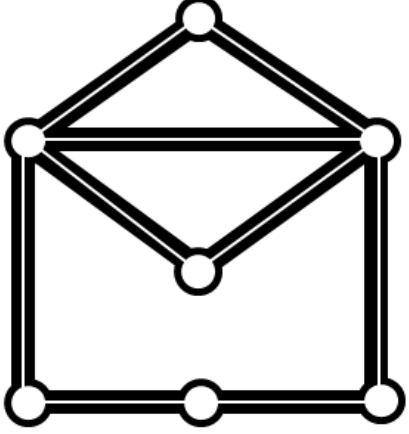
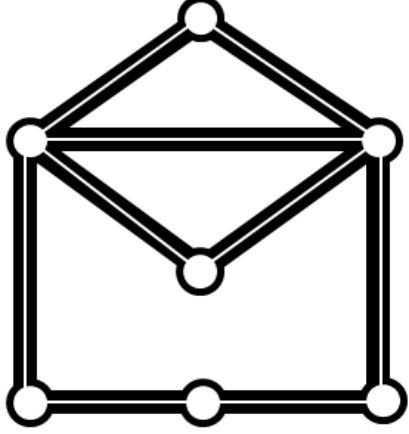
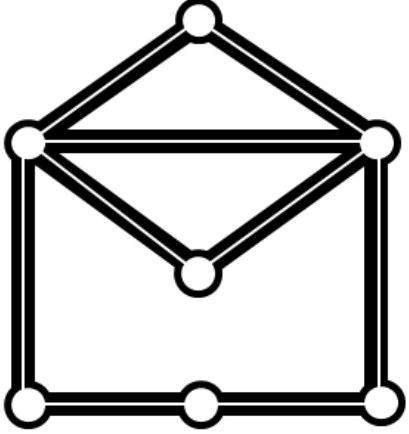
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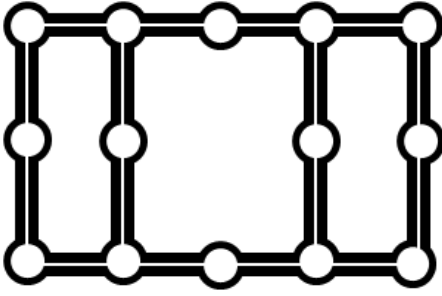
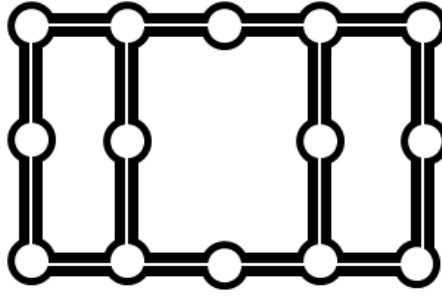
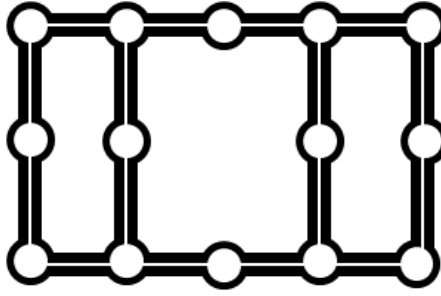
Path/Cycle/Neither



Path/Cycle/Neither



Path/Cycle/Neither



Path/Cycle/Neither