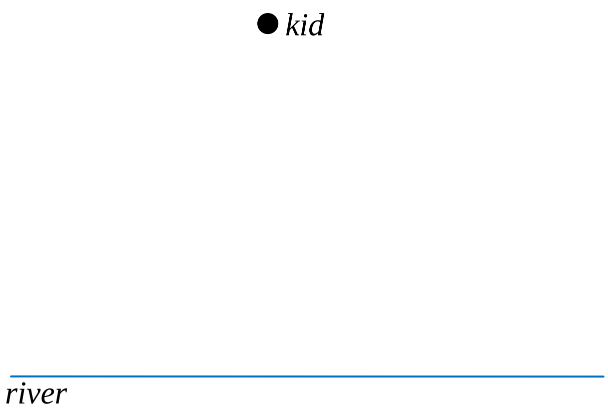
8. Distance, Altitude & Orthocenter

Problem 1. Suppose you have a river as a straight line and a kid who wants to run as quickly as possible to the river. Which route should he take? Draw several routes, decide which is the shortest, and describe its special geometric feature that distinguishes it from the other routes and makes it the shortest route.



The shortest route is ______________________

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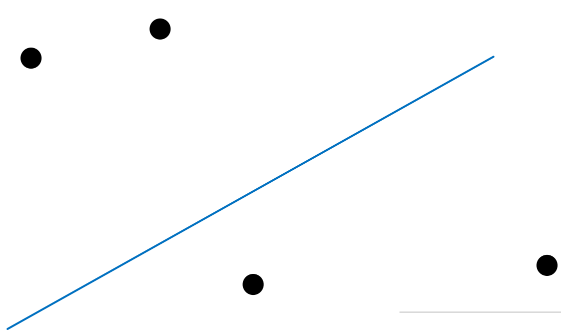
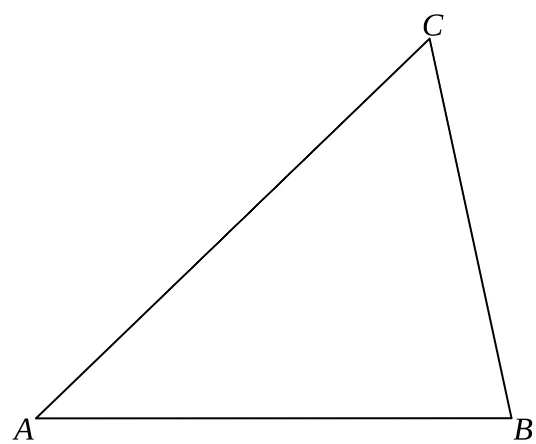
Problem 2. Now draw the shortest routes for all children in the next picture to the river. You are allowed to use for now any geometric drawing tool that you wish. Which tool is the most helpful here?



Theorem 1. The shortest route from a point *A* to a line *l* is the segment through *A* perpendicular to the line *l*. It practice, it can be drawn using a right triangle.

Definition 1. Drawing the shortest route from a point *A* to a line *l* is referred to as *dropping the perpendicular* from *A* to *l*.

Problem 3. In triangle *ABC* below drop the perpendicular from vertex *C* to side *AB*. How tall is triangle *ABC*?



Definition 1. The perpendicular from vertex *C* to line *AB* is called an *altitude* of triangle *ABC*.

Corollary 2. Three concepts are the same:
a) The *altitude* through vertex *C*;
b) The (shortest) *distance* from *C* to line *AB*;
c) The *perpendicular* from *C* to line *AB*.
So, if *C* were to walk to *AB*, the shortest route it would take would be the altitude.

Problem 4. Draw the other two altitudes of triangle *ABC* through vertex *A* and through vertex *C*.

Question 1. What do you notice about the three altitudes of triangle *ABC*? Is it a coincidence? Write down your conjecture.

The three altitudes in triangle *ABC*:

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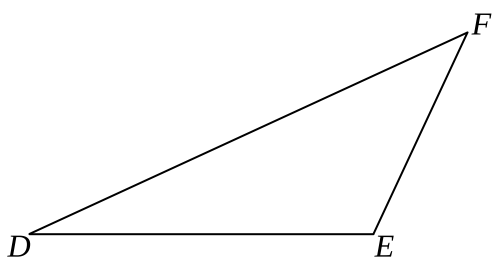
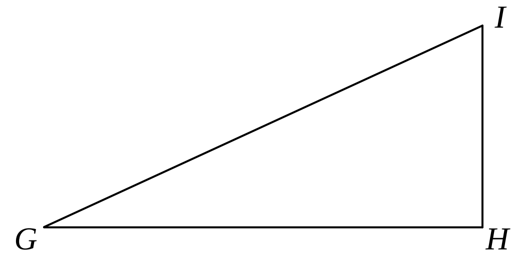
Problem 5. Check your conjecture on the triangles *DEF* and *GHI* below: draw the three altitudes for each triangle.



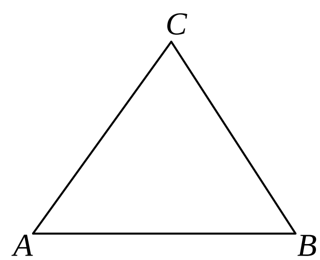
Hint 1. The altitude from *F* to line *DE* is hard to draw: extend segment *DE beyond E* to make a *line DE*. Then think of the shortest route that *F* would take to reach *line DE*: a perpendicular will have to be drawn from *F* to *line DE*.

Hint 2. The altitude from *D* to line *EF* may also give us trouble: extend segment *EF beyond E* to make a *line EF* and drop the perpendicular from *D* to *line EF*.



Hint 3. Two of the altitudes in the right triangle *GHI* are already drawn! Which are these two altitudes? Where is the third altitude?

Hint 4. Did you remember to extend the three altitudes for each triangle to see where they meet? Does your conjecture hold?



Question 2. Where did the altitudes intersect for triangles *ABC*, *DEF*, and *GHI*? Is this somehow related to the type of each triangle?

Theorem 2. The three altitudes in any triangle intersect in the same point, called the *orthocenter* of the triangle. The orthocenter is:

a) *Inside* the triangle if the triangle is *acute*.
b) *Outside* the triangle if the triangle is *obtuse*.
c) *Coincides* with the vertex of the right angle is the triangle is right.

RECAP 1: New Vocabulary and Ideas

Check ALL correct answers. Explain your choice and provide details.

1. Which of the following are special *centers* for a triangle?
   - I. The intersection of the *medians* in a triangle.
   - II. The intersection of the *midsegments* of a triangle.
   - III. The intersection of the *altitudes* of a triangle.
   - IV. The intersection of the *diagonals* of a triangle.
   - V. The intersection of the *perpendicular bisectors* of the sides of a triangle.
2. To *drop a perpendicular* means:
   - I. To drop your right triangle.
   - II. To draw a perpendicular from a point to a line.
   - III. To go vertically down from a point.
   - IV. To draw the shortest route from a point to a line.
   - V. Half of the above.
3. The sentence "The orthocenter is *outside* your triangle.":
   - I. Refers to a triangle *without* an orthocenter.
   - II. Is true for *every* triangle.
   - III. Makes *no* sense.
   - IV. Indicates that the triangle is *right*.
   - V. Implies there is an *obtuse* angle somewhere.

RECAP 2: Matching Words

Connect each word or part of a word on the left with its all possible continuations on the right and explain the meaning of the resulting word or expression.

medi	point
mid	tude
atti	pedic
ortho	dontist
ampli	bisector
alti	center
circum	segment
apti	gonal
perpendicular	dox
comedi	scribed
magni	an

RECAP NOTE 3: Origin and synonyms

"Ortho" comes from the Greek *orthos*, which means "straight, right". *Orthogonal* is a synonym of *perpendicular*: "An altitude in a triangle is orthogonal to the opposite side".

RECAP 4: Theory

We have seen so far that

- the three medians in a triangle,
- the three perpendicular bisectors of the sides of the a triangle, and
- the three altitudes in a triangle

always intersect in a point. These points are called respectively:

- the *centroid* (or the medicenter),
- the *circumcenter* (or the center of the circumscribed circle about the triangle), and
- the *orthocenter* of the triangle.