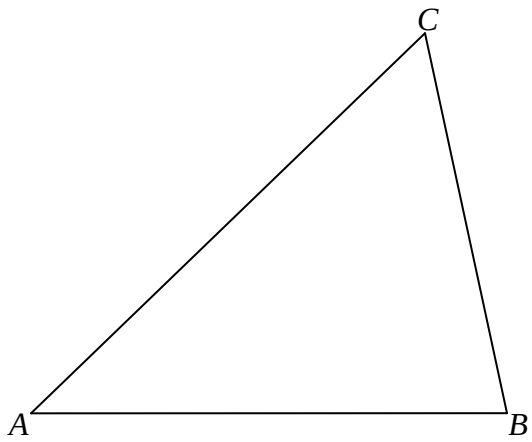
1. Centroid of a Triangle

Problem 1. Find the *midpoints* of sides *AB*, *BC*, and *CA* and label them C_1, A_1, and B_1. Connect each midpoint to the opposite vertex of the triangle.



Definition 1. The segments AA_1, BB_1, and CC_1 are called the *medians* of triangle *ABC*. (Every triangle has three medians.)

Problem 2. Draw on your graph paper three more triangles (to make things simple, make sure that the vertices of the triangles are at the grid points):

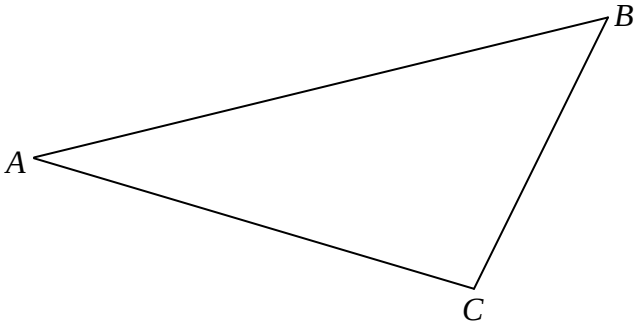
- a right triangle *DEF*,
- an isosceles triangle *GIH*, and
- an obtuse triangle *JKL*.

Draw the three medians in each triangle.

(*Hint:* The grid paper will help you locate the midpoints of the sides practically without using a ruler. How is that?)

Question 1. What do you notice about the three medians in each triangle? Do you think it is a coincidence? What is your conjecture? Write it down.

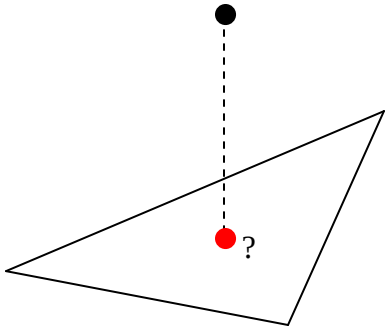
Problem 3. Test your conjecture on the triangle below. Does it still hold?



Theorem 1. The three medians in a triangle (always!) intersect in a single point.

Definition 2. The point where the medians of a triangle intersect is called the *centroid* of that triangle. Other names for this famous point are: *center of mass* and *medicenter* of the triangle.

Question 2. Can you guess where the name *"center of mass"* comes from? Experiment with a rigid model of a triangle (e.g., a triangle made of cardboard or metal): try to hang the triangle on a string from a point on the triangle. From which point can you hang the triangle so that it is *horizontal*?



Question 3. Let M be the centroid of any of the triangles on page 1. Along each median, measure how much further M is from the vertex than from the opposite midpoint; i.e., how much are the *ratios:*

- $AM : MA_1$?
- $BM : MA_1$?
- $CM : MC_1$?

What is your conjecture?

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Problem 4. Test your conjecture on the other triangles on page 1 and your graph paper. Does the conjecture still hold?

Theorem 2. The centroid divides each median in ratio 2:1 counted from the vertex. In other words,

- $AM : MA_1 = 2 : 1$
- $BM : MA_1 = 2 : 1$
- $CM : MC_1 = 2 : 1$.

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RECAP 1: New Vocabulary

What new words (concepts) did you learn in this unit? List them below and explain what they mean.

RECAP 2: New Theory

What new facts did you learn about

- the medians in a triangle?
- the centroid of a triangle?

RECAP 3: Mathematical Logic

- Did we *prove* any of the new facts? How are we so sure they are true?
- What is the difference between a *conjecture*, a *theorem*, and a *definition?* Which needs to be proven?