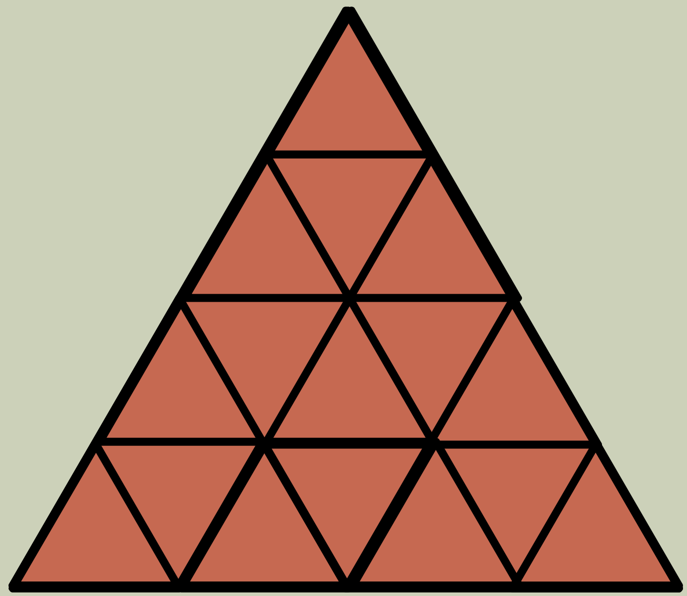
2^{ND} CLASS WARM-UP 9/9/14

Part 1: (Mild) How many triangles are in this image?

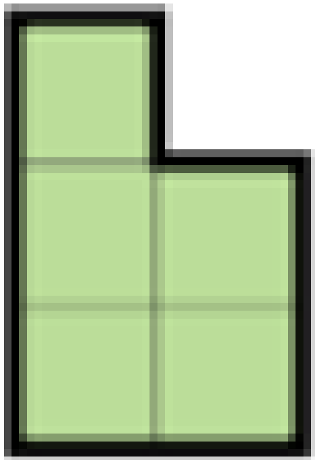


Part 2: (Spicy) What if you add more rows of triangles?

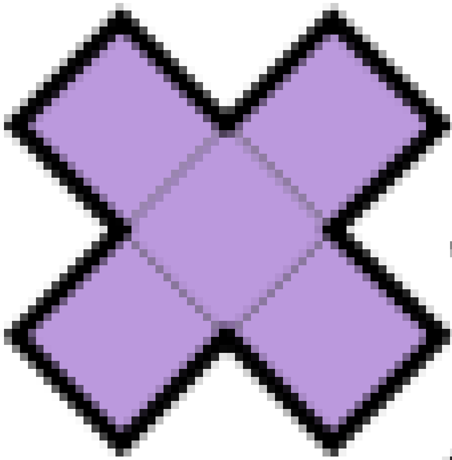
How many triangles would a 100-row picture have?

FIRST CLASS WARM-UP 9/2/14

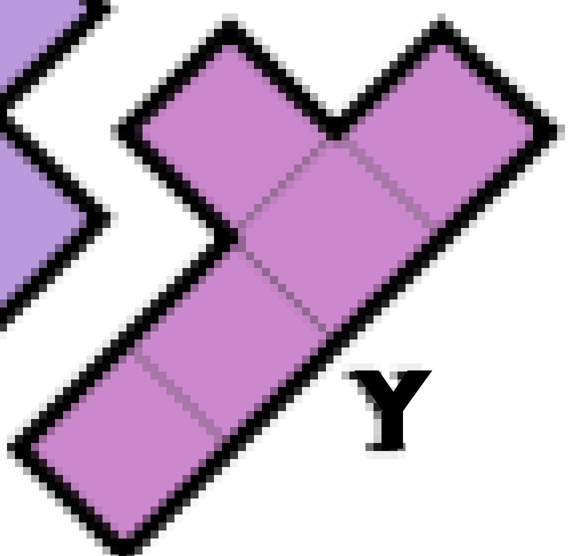
IS IT POSSIBLE TO COVER THIS 5X5 GRID USING ONLY ONE TYPE OF PENTOMINO? WHAT ABOUT A COMBINATION OF B, X, AND Y PENTOMINOES?



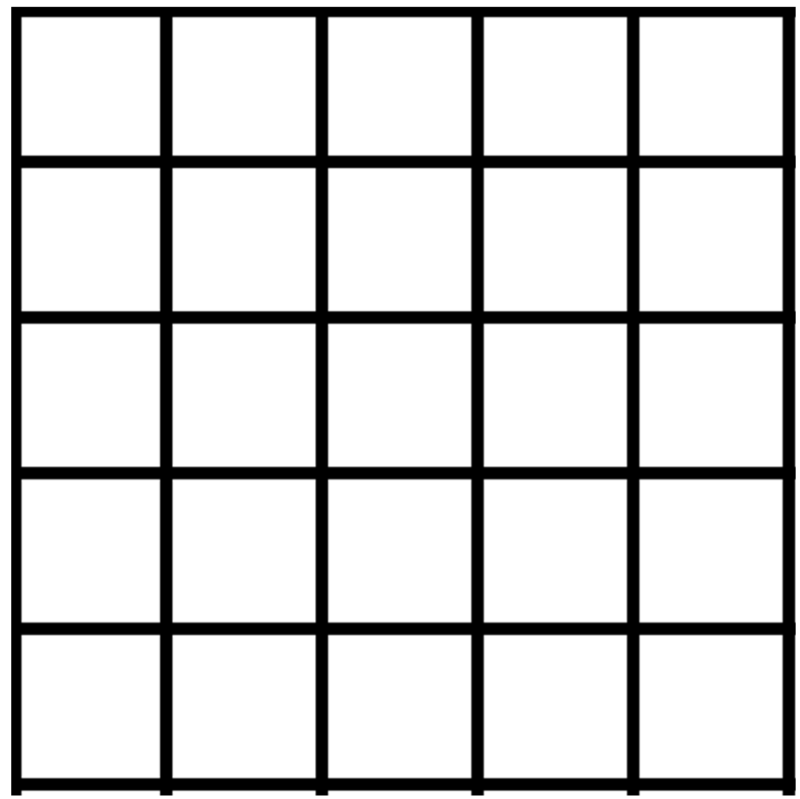
b



X



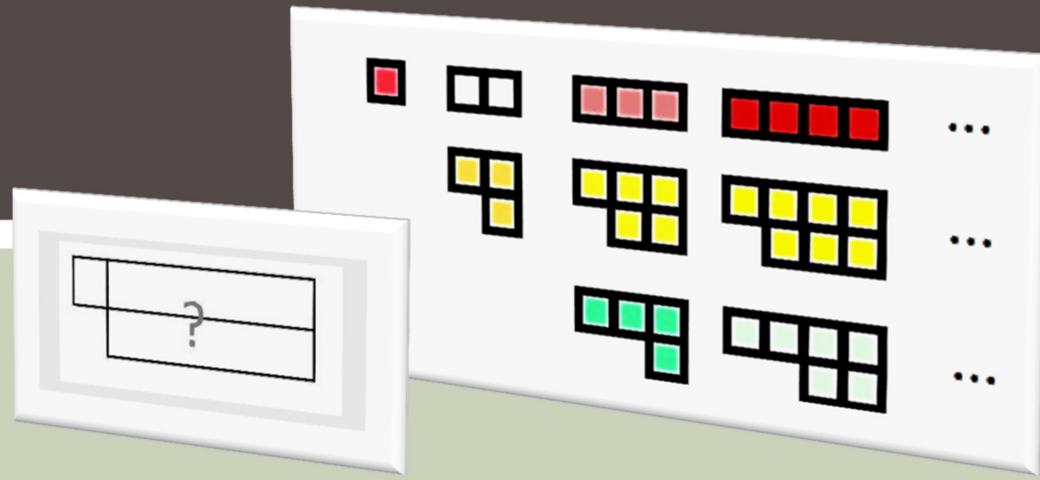
Y



COMBINATORICS I

An unsolved tiling puzzle

AGENDA

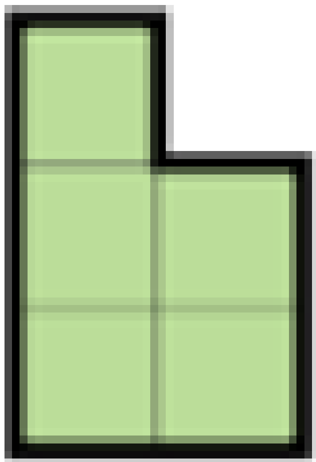


- Introductions
- Challenges
  - (warmup) Tiling with b, x, and y pentominoes
  - Tiling with dominoes on a chess board
  - Tiling with dominoes on a plank
  - Counting border patterns
  - An unsolved tiling puzzle
- Problem Solving Strategies
- Generalizing/Digging into Challenges

HI! I'M ZANDRA VINEGAR

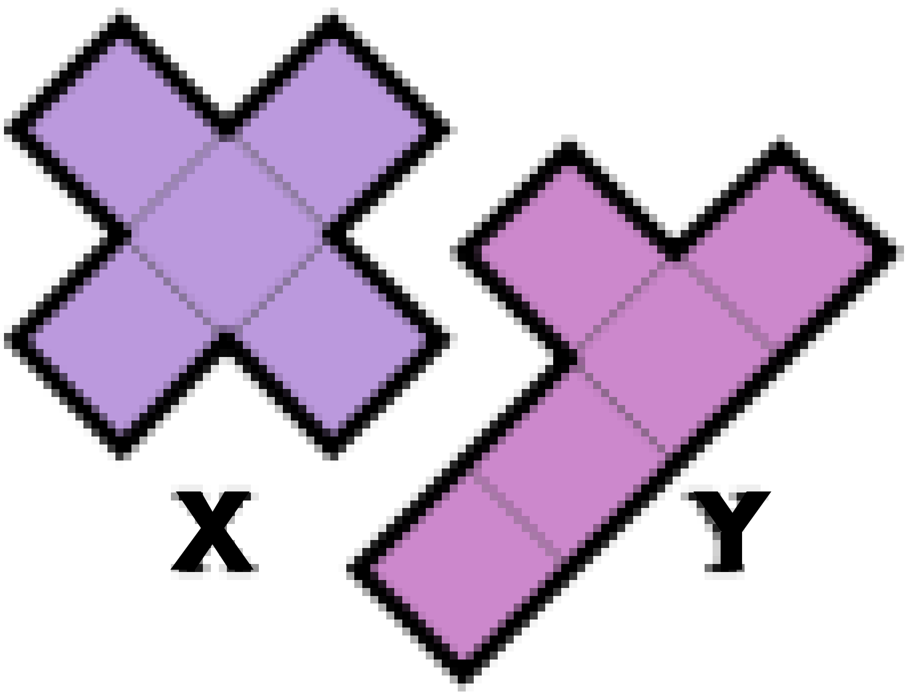
HI! I'M ZANDRA VINEGAR

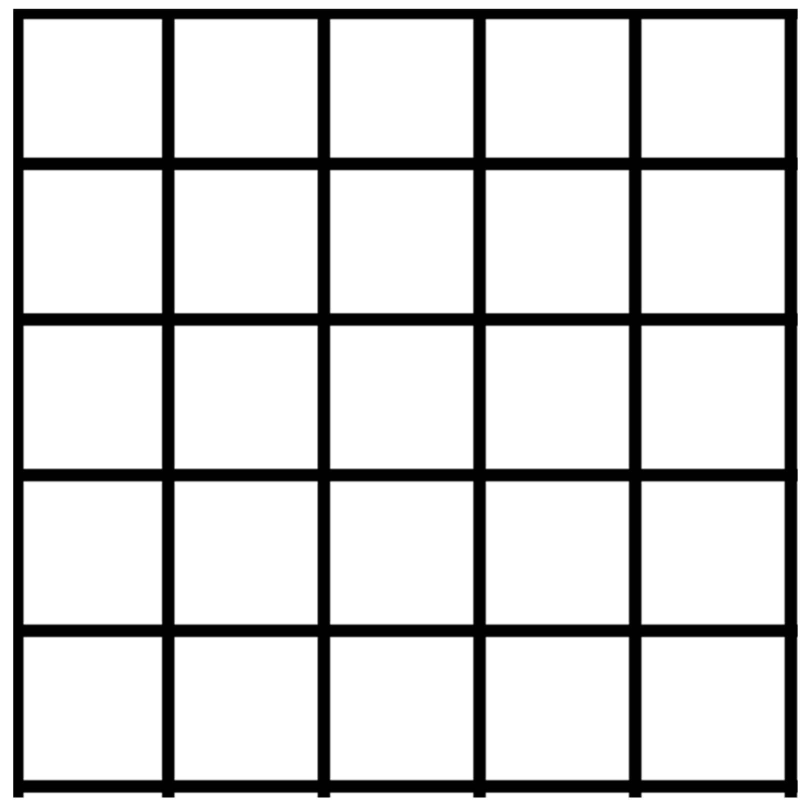
- Graduate of MIT
  - Major: Mathematics (especially TCS)
  - Cert: Student Teaching @ Another Course to College HS & CRLS HS
- 1 Year at MoMath (the Museum of Mathematics)
  - Manhattan NY – (yes! You should visit!)
  - K-12 Education: teaching 45 minute sessions to school groups, curriculum development, social media
- This past summer @ SPMPS (the Summer Program in Mathematical Problem Solving)
  - Underserved MS Students from NYC public schools
  - "Cryptography and Hamming Codes" & "Sharing Math with the World"
- Currently
  - Math Circles (Berkeley + Stanford)
  - AoPS (Online School)
  - Proof School (Volunteer)
- Hobbies
  - Making Art
  - Building Furniture
  - Cooking
  - Hiking
  - Reading (audio books)



b

IS IT POSSIBLE TO COVER THIS 5X5 GRID USING ONLY ONE TYPE OF PENTOMINO? WHAT ABOUT A COMBINATION OF B, X, AND Y PENTOMINOES?





HOW MANY DIFFERENT SOLUTIONS ARE THERE?

Only x:

Only y:

Only b:

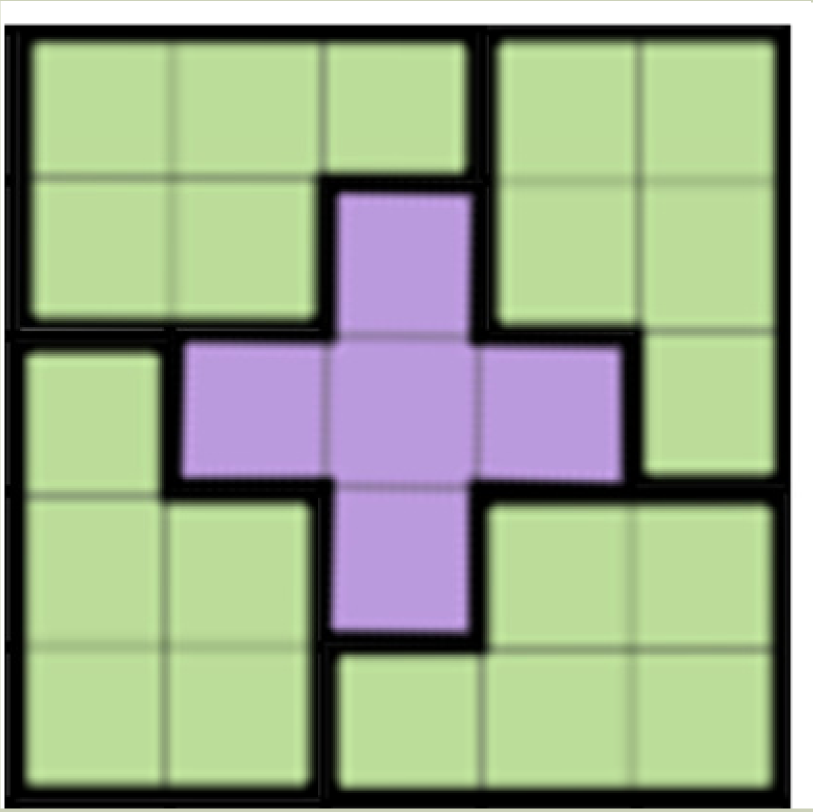
b & x:

x & y:

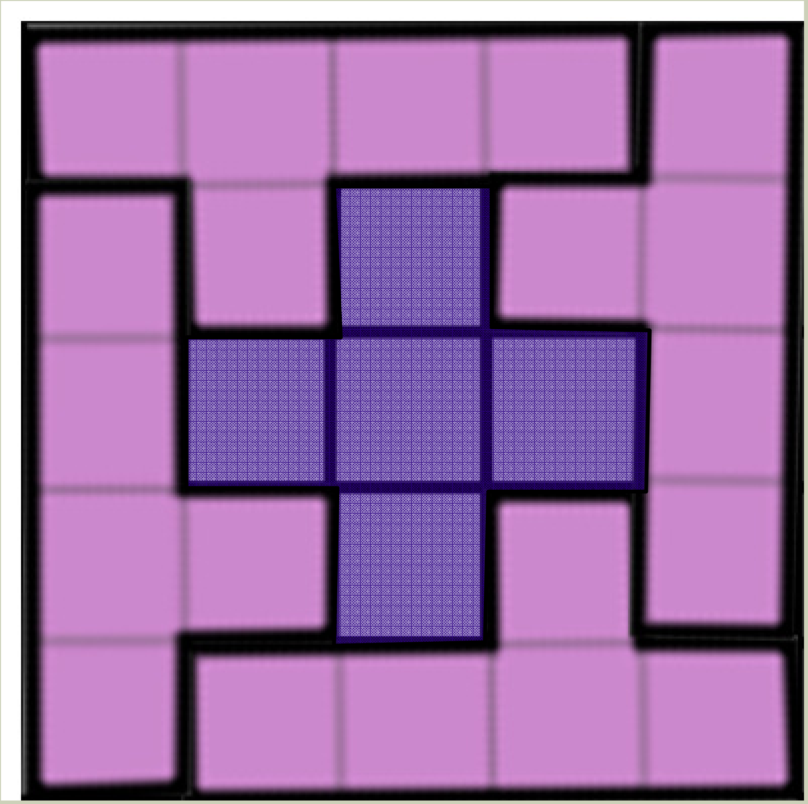
y & b:

b, x, & y:

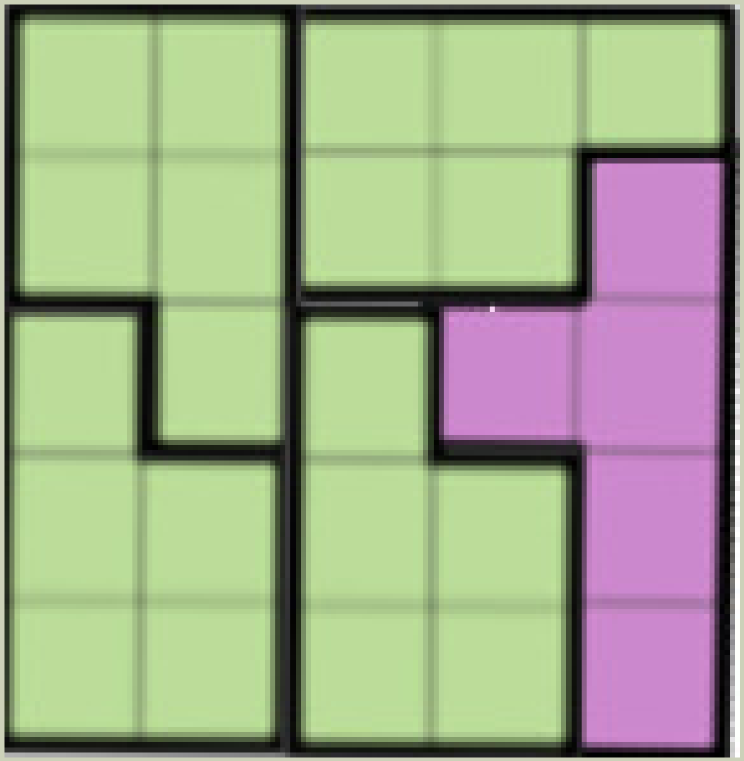
SOLUTION



SOLUTION

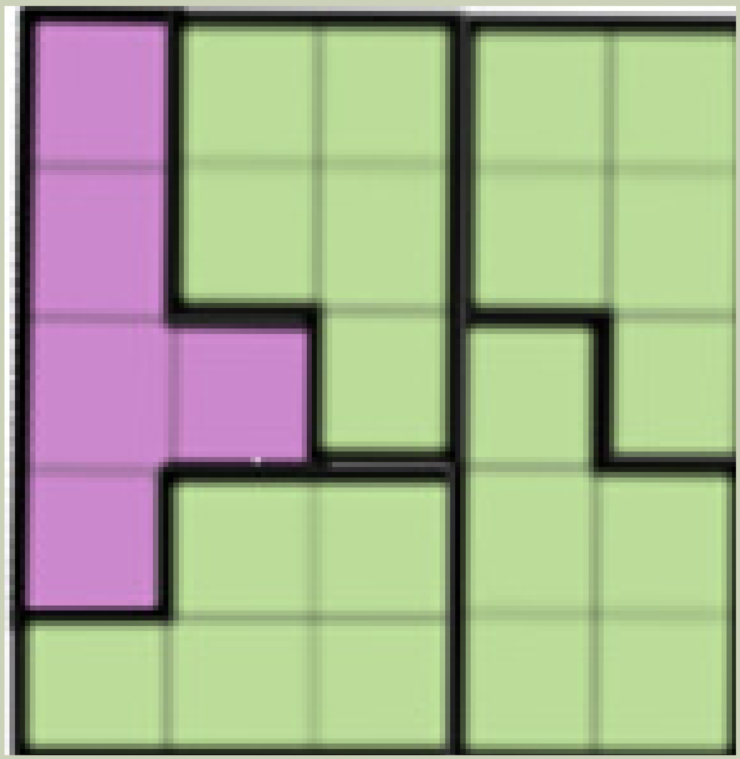


SOLUTION:

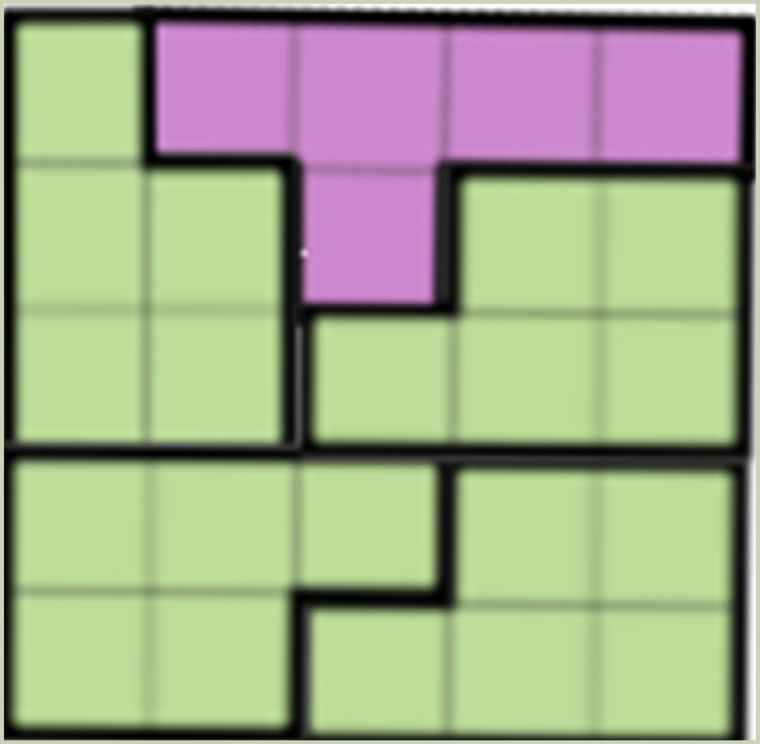


SOLUTION:

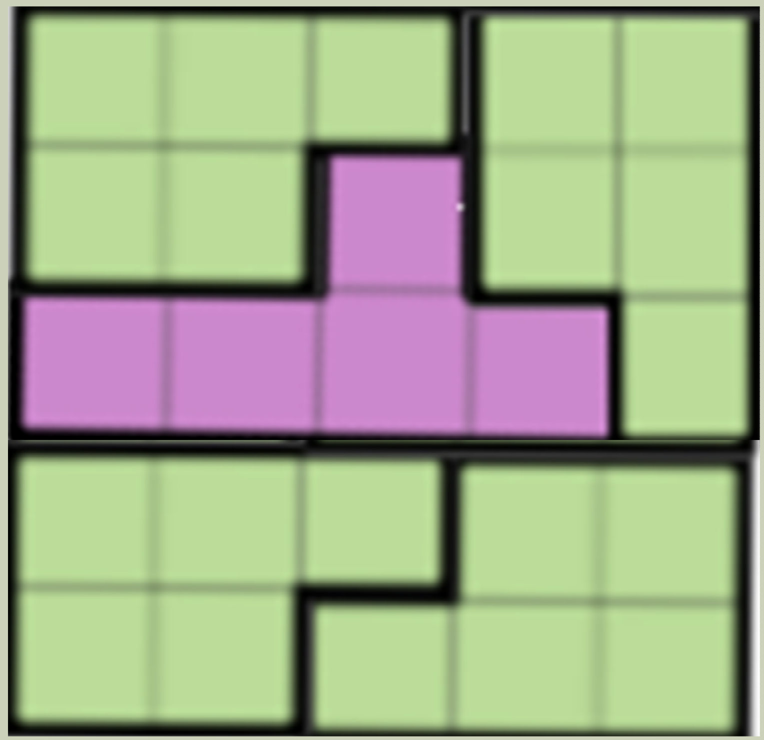
SOLUTION:



SOLUTION:



SOLUTION:



HOW MANY DIFFERENT SOLUTIONS ARE THERE?

Only x: 0

Only y: 0

Only b: 0

b & x: 1

x & y: 1

y & b: 8 (2 x 4 rotations)

b, x, & y: 0

GREAT JOB!

I think you deserve a reward. Here is a fuzzy puppy

- YOUR IDEAS HERE

SUMMARY

Problem Solving Strategies

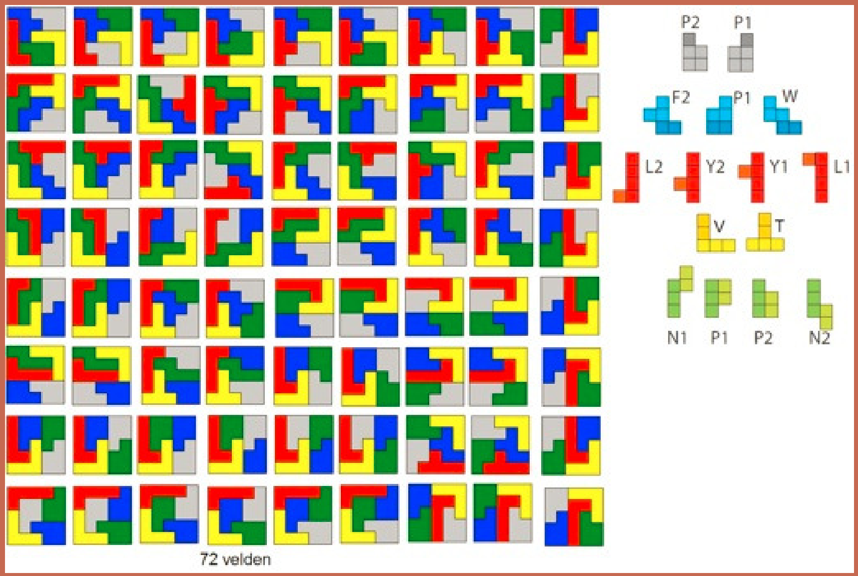
Solutions & Proofs

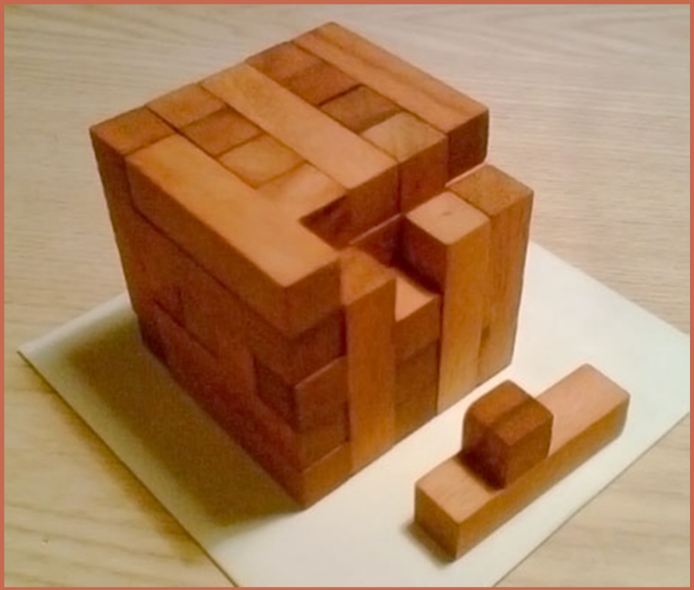
Digging Deeper: Ways to Generalize the Problem

DIGGING DEEPER

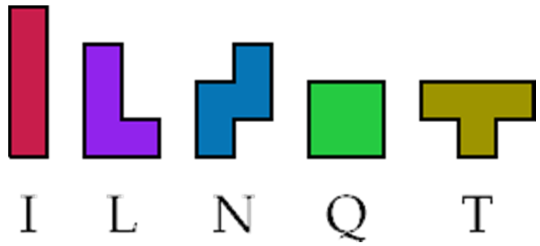
Game Design
(Penguins on Ice)

3D?!

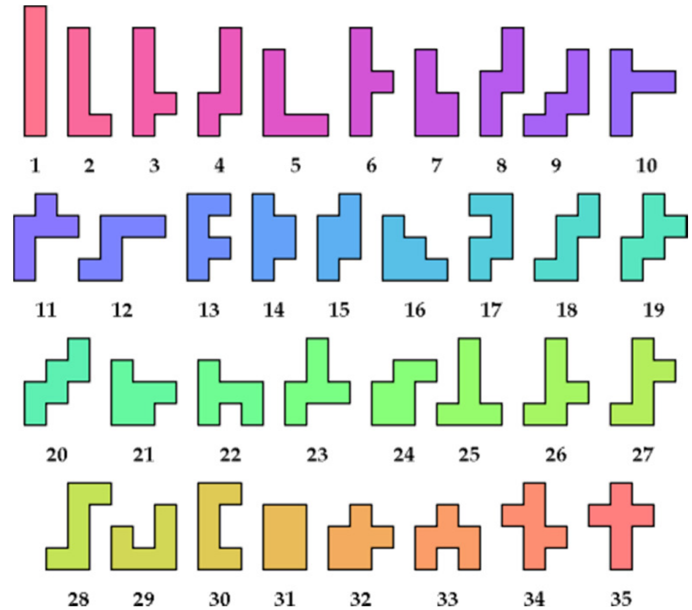




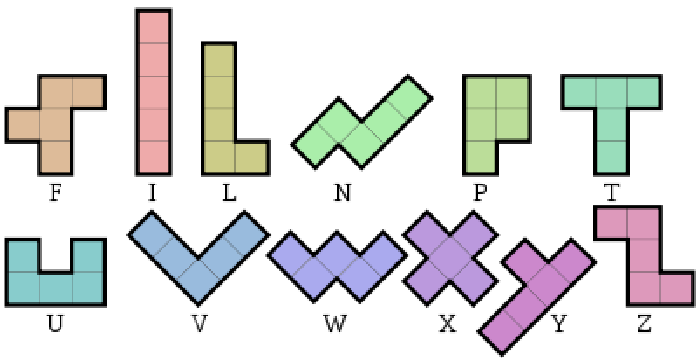
Tetrominoes



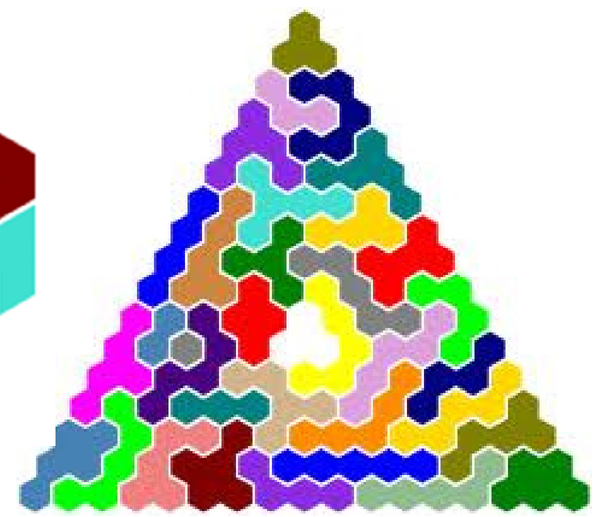
Hexomiones



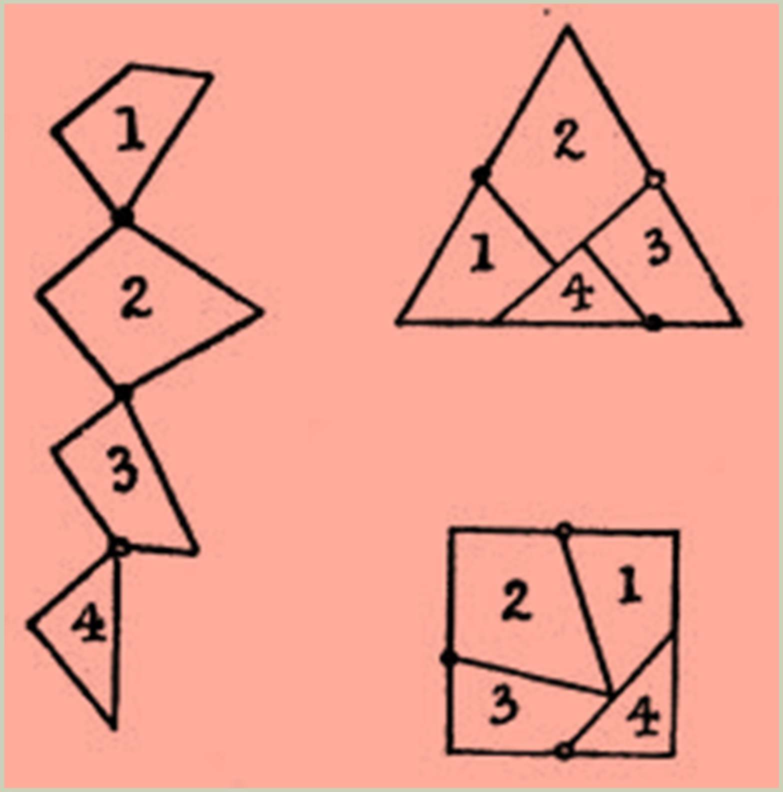
PENTOMIONES

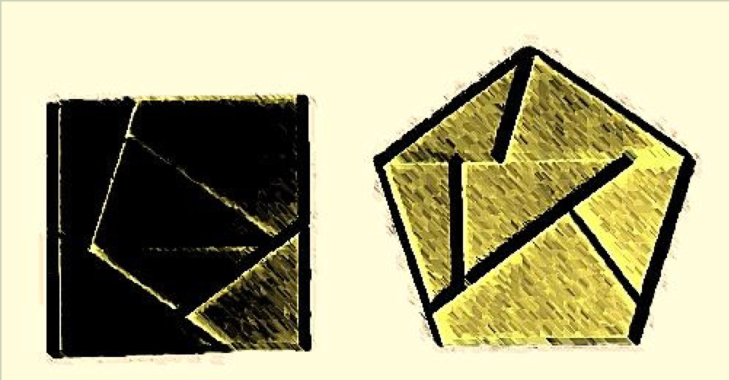


...also, triangular pieces,
Called polyaminoes
And polyhexes

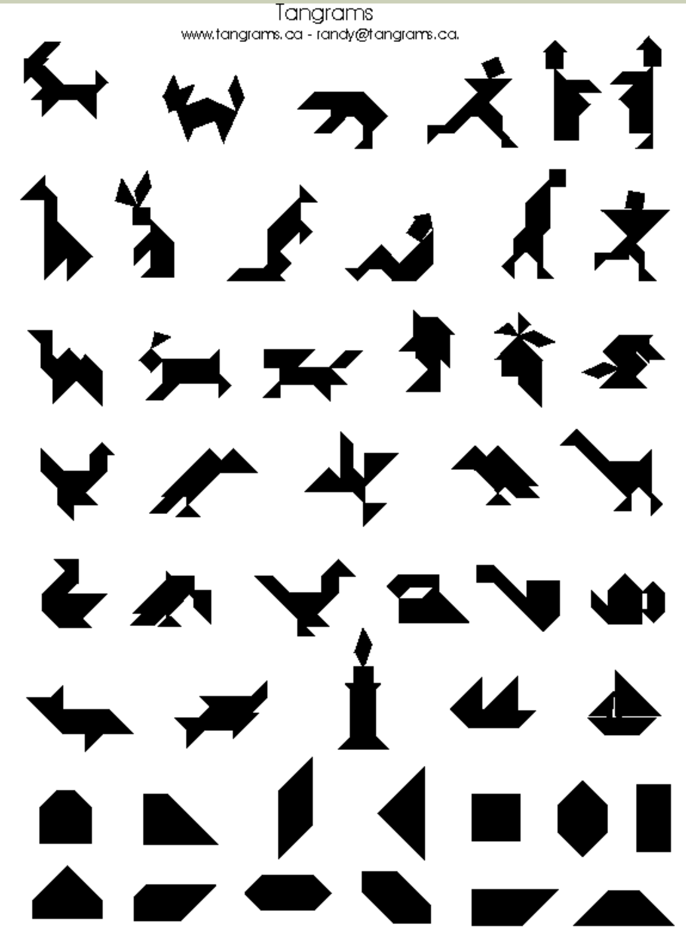


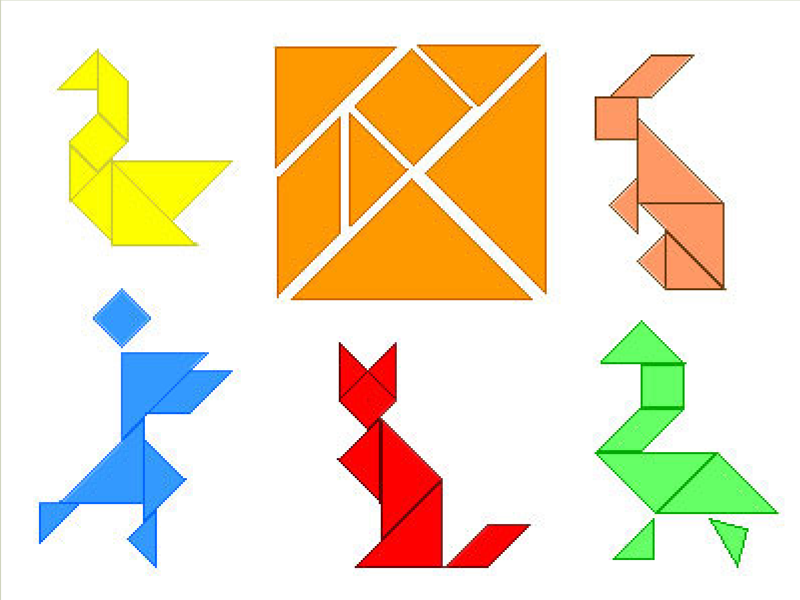
HABERDASHER'S PUZZLE





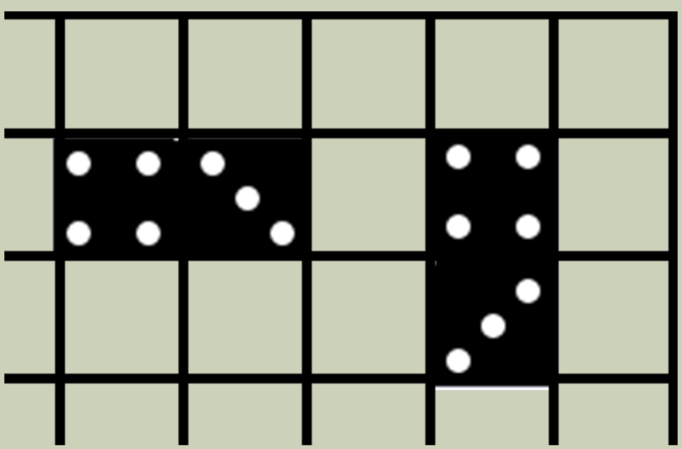
TANGRAMS

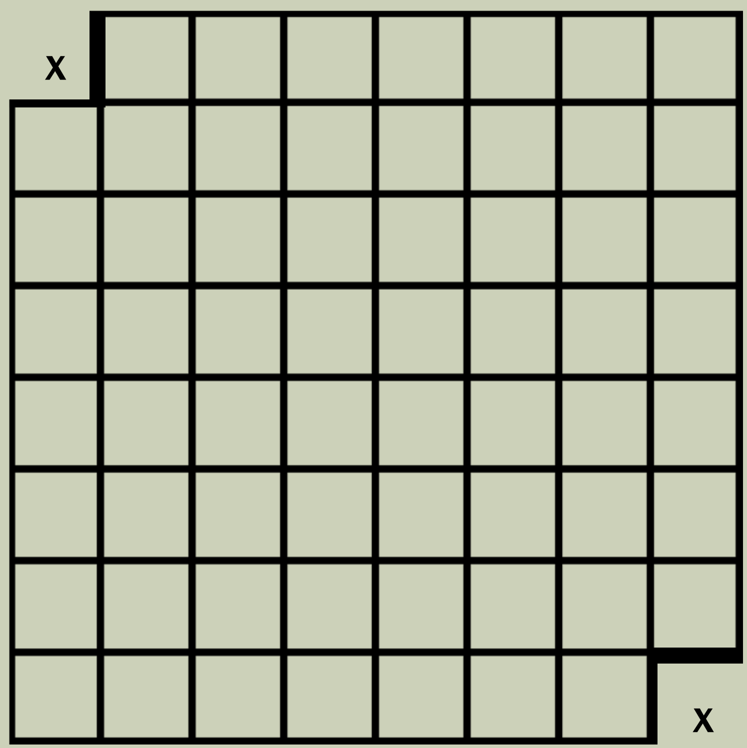




IS IT POSSIBLE TO COVER AN 8X8 BOARD WITH DOMINOES IF TWO 1X1 SQUARES HAVE BEEN CUT OUT OF OPPOSITE CORNERS OF THE BOARD

Each domino covers two adjacent squares:

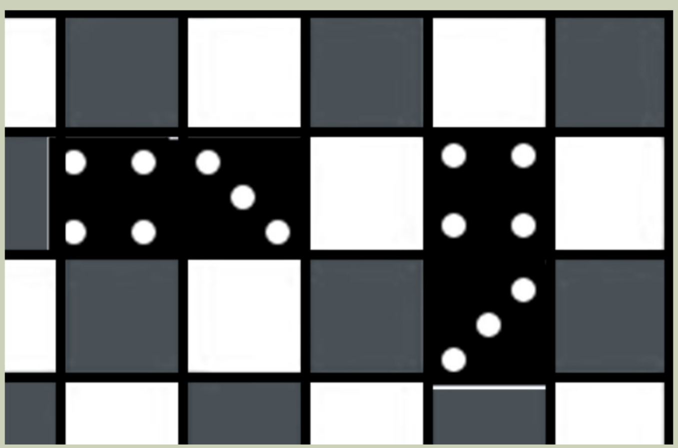


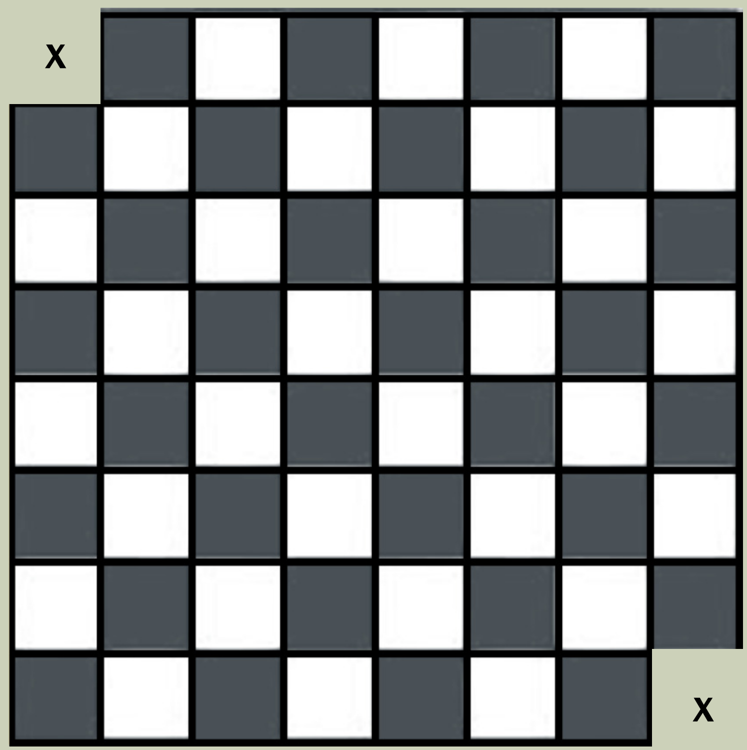


SOLUTION

No, it's not possible.

Each domino covers two adjacent squares:

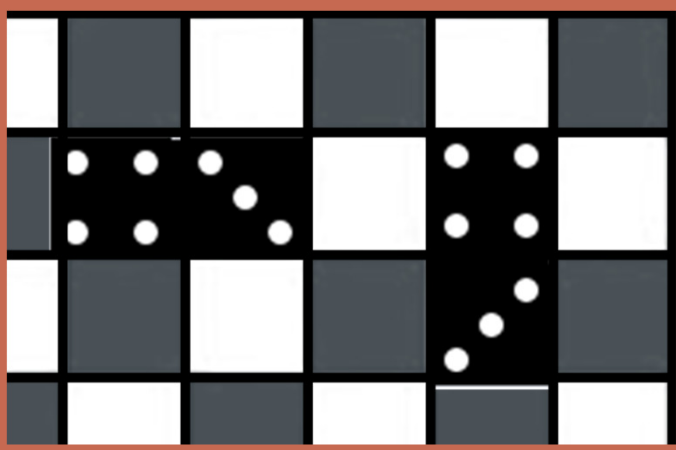


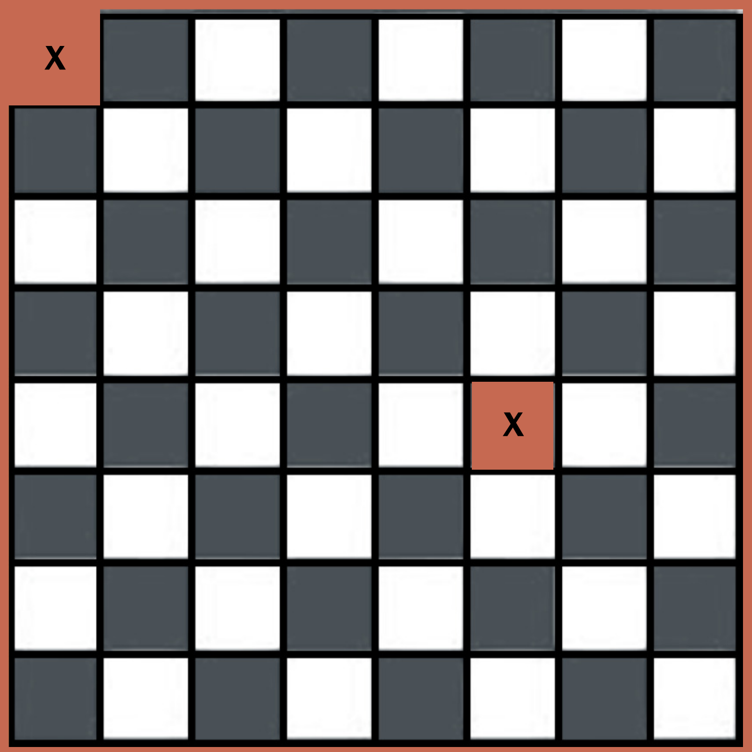


DIGGING DEEPER

When is it possible?

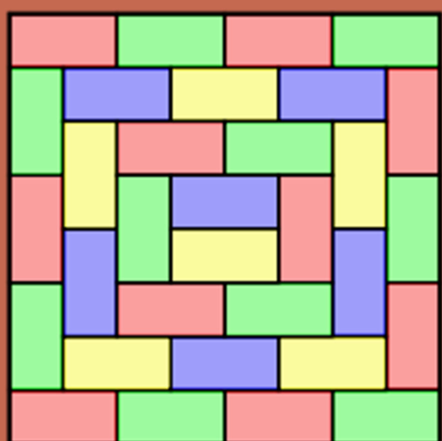
Each domino covers two adjacent squares:

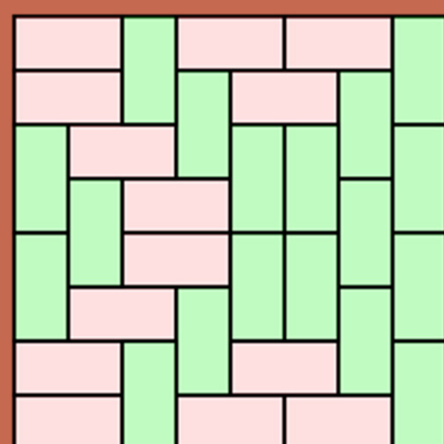


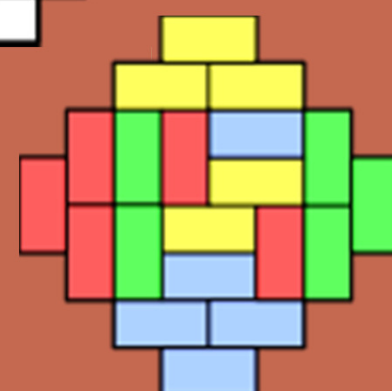


COMBINATORICS

In how many ways is it possible?



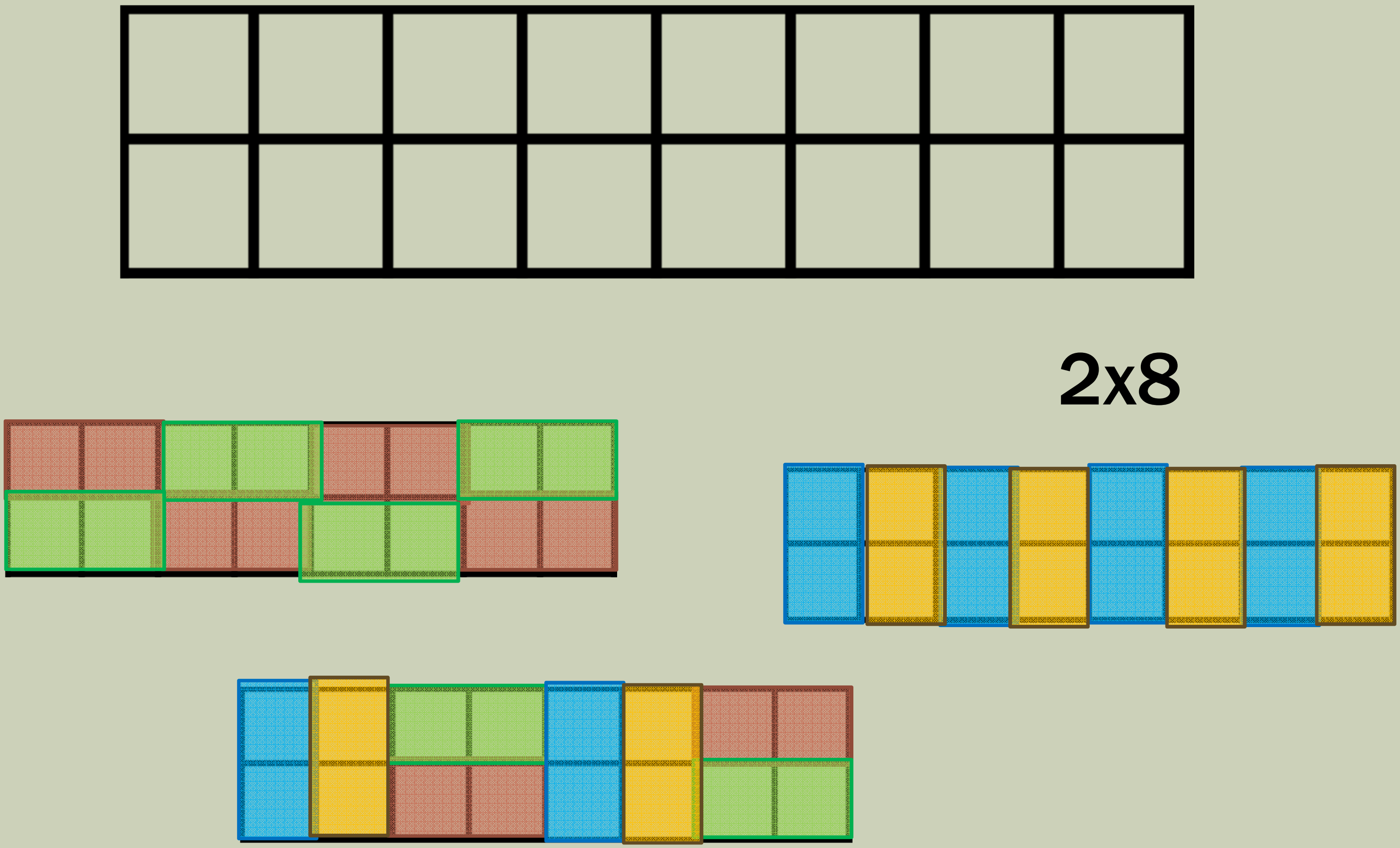




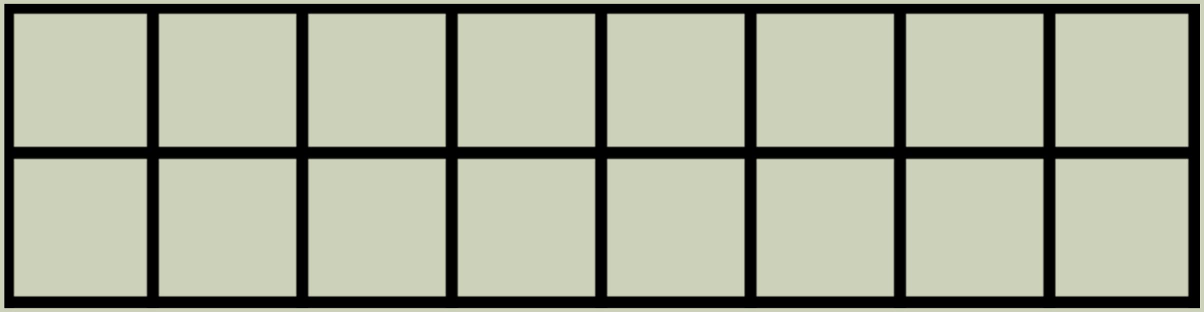
BREAK TIME!

This is a red panda! ☺

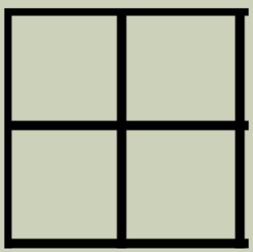
HOW MANY WAYS ARE THERE TO TILE A 2X8 RECTANGLE WITH 2X1 DOMINOES?



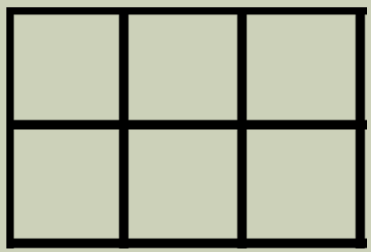
HOW MANY WAYS ARE THERE TO TILE A 2X8 RECTANGLE WITH 2X1 DOMINOES?



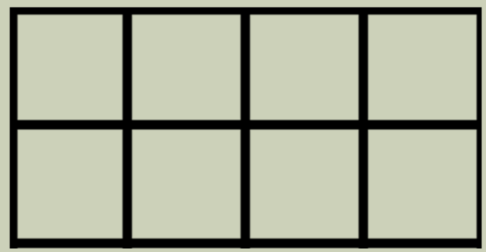
2x8



n=2

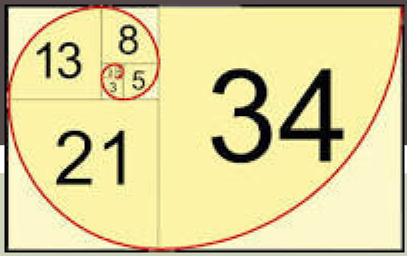


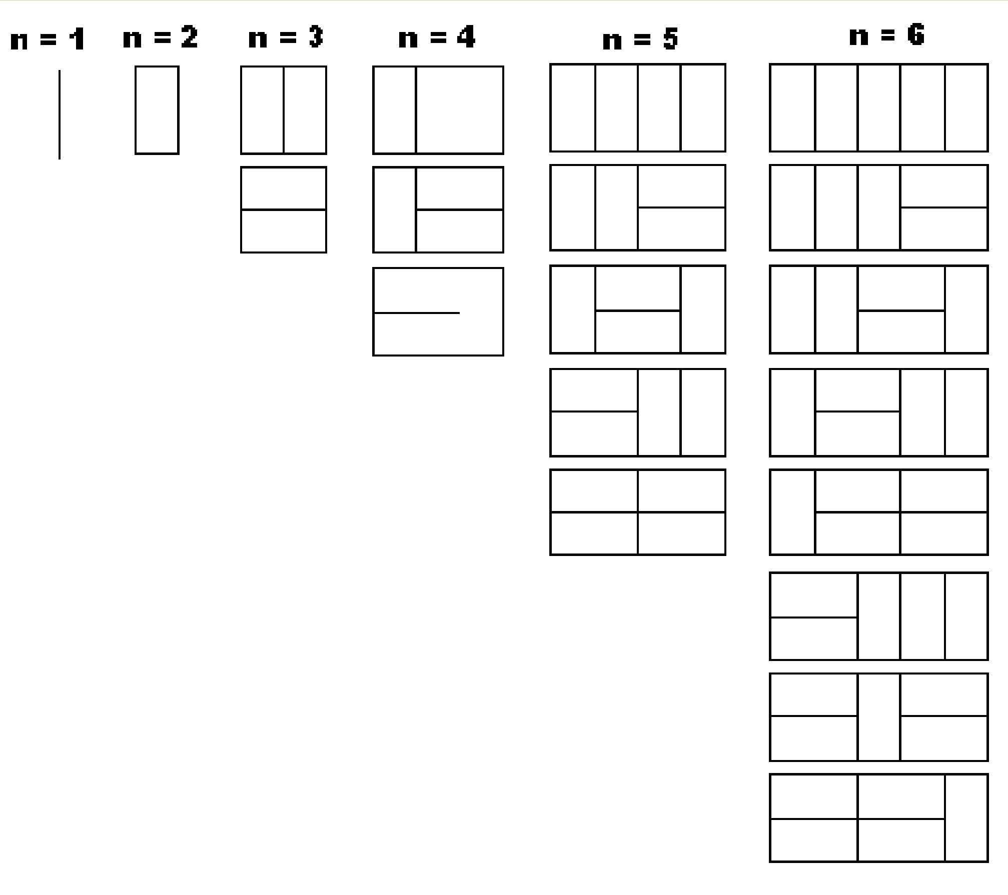
n=3



n=4

SOLUTION

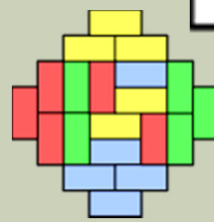


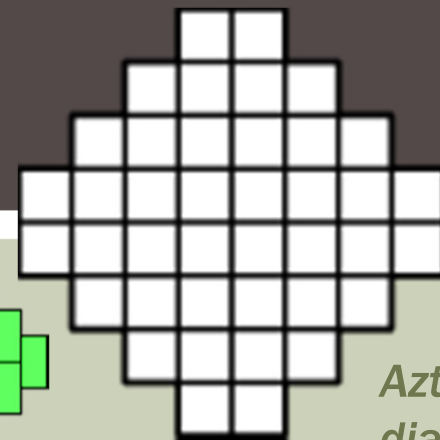


FIBONOCCI

- The Fibonacci Sequence is the series of numbers:
- 1, 1, 2, 3, 5, 8, 13, 21, 34, ...
- The next number is found by adding up the two numbers before it.

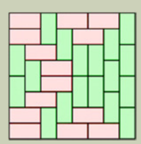
WRAP UP





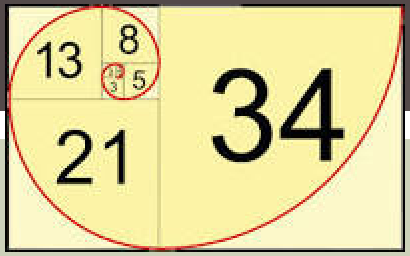
Aztec diamond

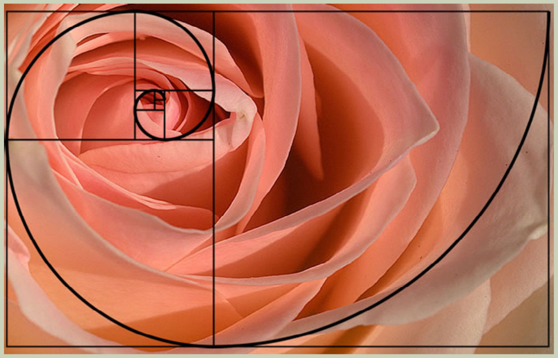
- Problem Solving Strategies
- Solutions & Proofs
- Ways to Generalize the Problem
  - The number of ways to cover an nxm rectangle with dominoes was calculated independently by [Temperley & Fisher (1961)] and [Kasteleyn (1961)]
  - Squares are a special case.

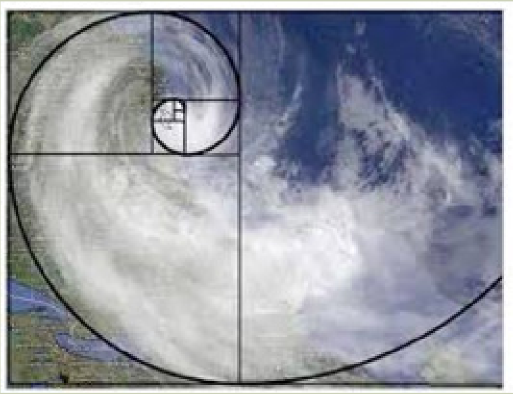


Also, Congratulations! You've all earned this Pink Fairy Armidillo!

FIBONACCI







- Is there a way to jump straight to the n^{th} Fibonacci number?

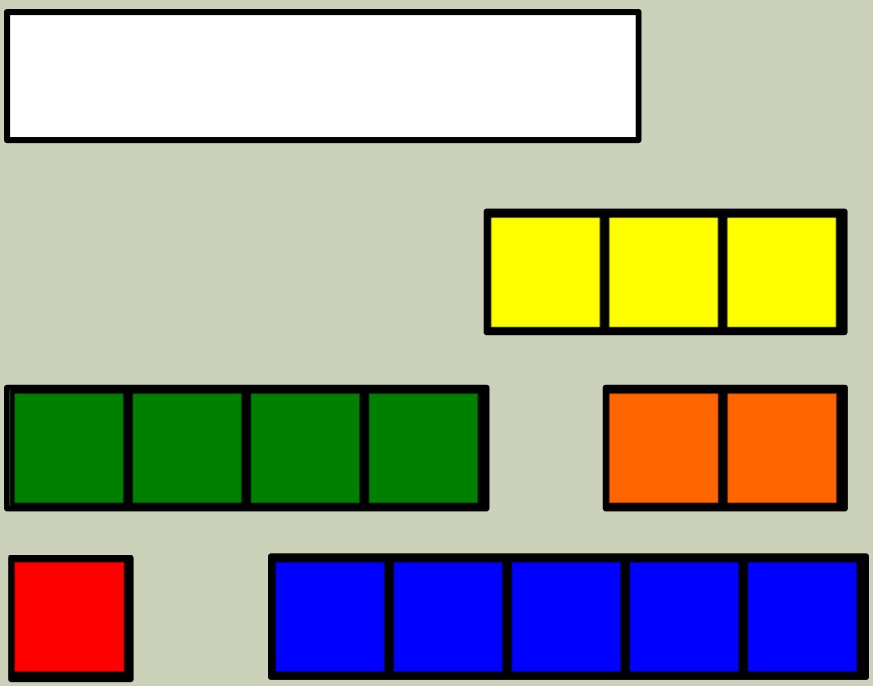
Yes:

$$\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n-\left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

- http://www.mathsisfun.com/numbers/fibonacci-sequence.html
- http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibFormula.html

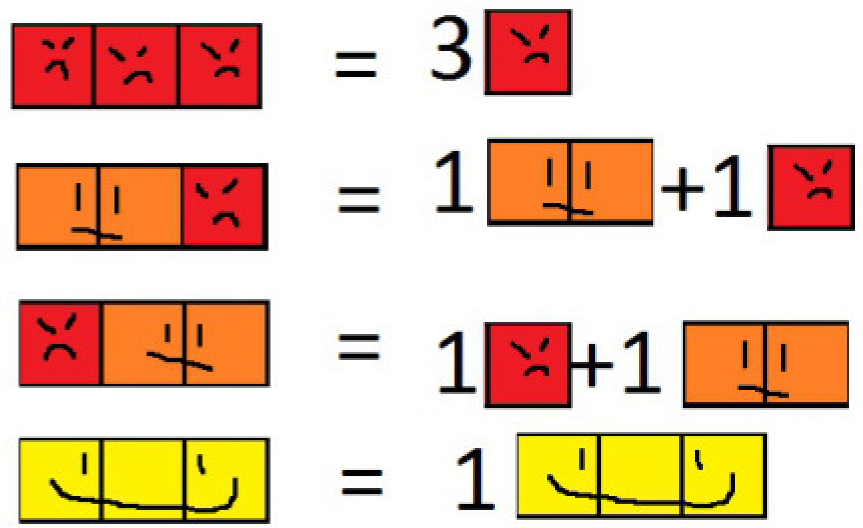
COUNTING BORDER PATTERNS

How many ways are there to tile this 1x5 rectangle with any of the 5 types of colored tiles below?

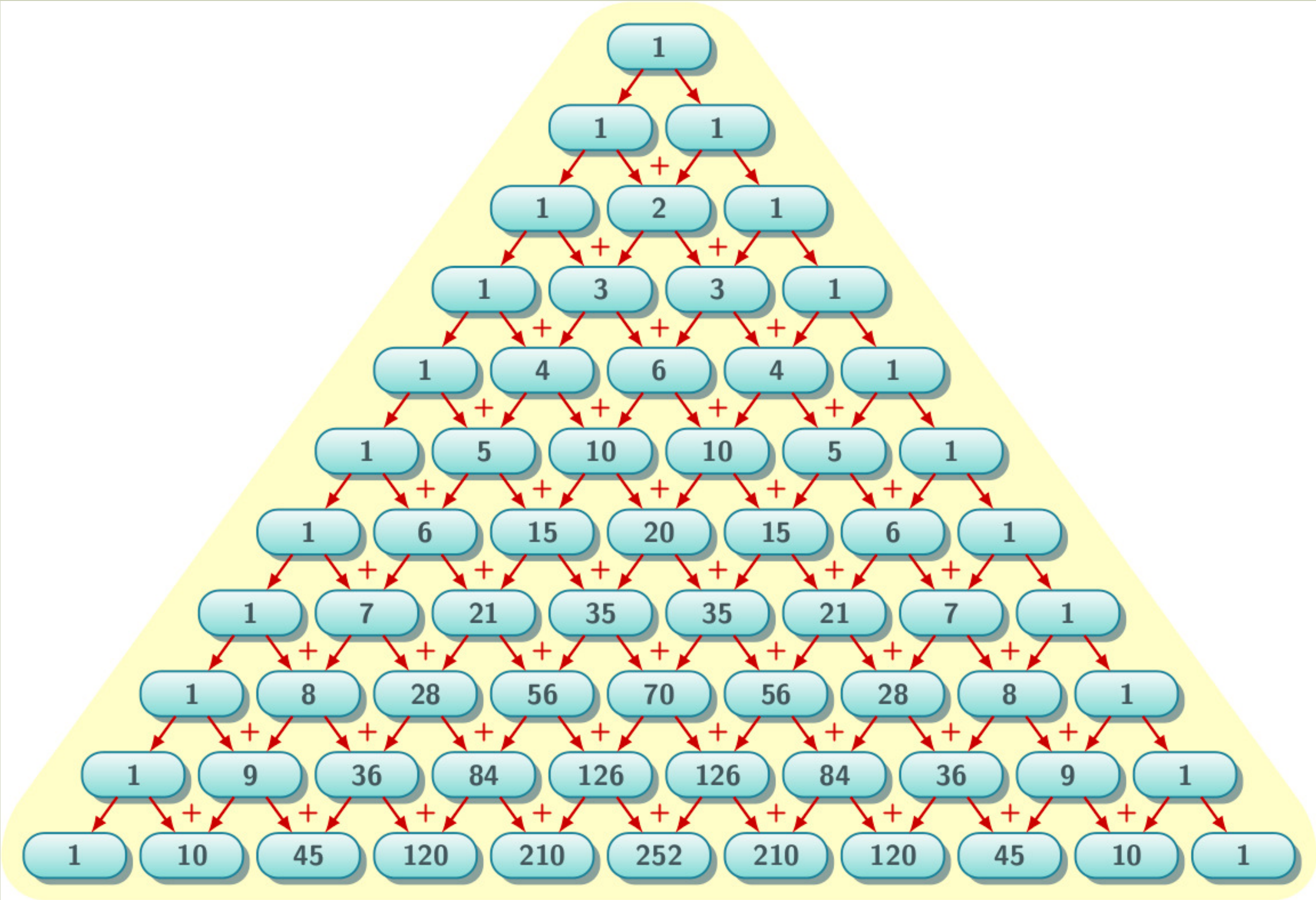


- This puzzle is from the blog “Baking and Math”
- The related unsolved problem is from a presentation by Pamela Harris at the annual Midwest Women in Mathematics Symposium
- http://bakingandmath.com/2014/04/25/open-problem-in-combinatorics-tiling-a-floor-no-background/

1X3 TILING



PASCAL'S TRIANGLE



BINOMIAL COEFFICIENTS

$$(a+b)^0 = 1$$
$$(a+b)^1 = 1a + 1b$$
$$(a+b)^2 = 1a^2 + 2ab + 1b^2$$
$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$$
$$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$$
$$(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$$

BINOMIAL COEFFICIENTS

row (0): 1 → $\frac{0!}{0!}$

row (1): 1 1 → $\frac{1!}{0!1!}$ $\frac{1!}{1!0!}$

row (2): 1 2 1 → $\frac{2!}{0!2!}$ $\frac{2!}{1!1!}$ $\frac{2!}{2!0!}$

row (3): 1 3 3 1 → $\frac{3!}{0!3!}$ $\frac{3!}{1!2!}$ $\frac{3!}{2!1!}$ $\frac{3!}{3!0!}$

row (4): 1 4 6 4 1 → $\frac{4!}{0!4!}$ $\frac{4!}{1!3!}$ $\frac{4!}{2!2!}$ $\frac{4!}{3!1!}$ $\frac{4!}{4!0!}$

row (5): 1 5 10 10 5 1 → $\frac{5!}{0!5!}$ $\frac{5!}{1!4!}$ $\frac{5!}{2!3!}$ $\frac{5!}{3!2!}$ $\frac{5!}{4!1!}$ $\frac{5!}{5!0!}$

row (n): 1. . . . . .1 → $\frac{n!}{0!n!}$ $\frac{n!}{1!(n-1)!}$ . . . . $\frac{n!}{(n-1)!1!}$ $\frac{n!}{n!0!}$

Row n

Position k

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

SOLUTION

$$1 = 2^0 = 1$$

$$1 + 1 = 2^1 = 2$$

$$1 + 2 + 1 = 2^2 = 4$$

$$1 + 3 + 3 + 1 = 2^3 = 8$$

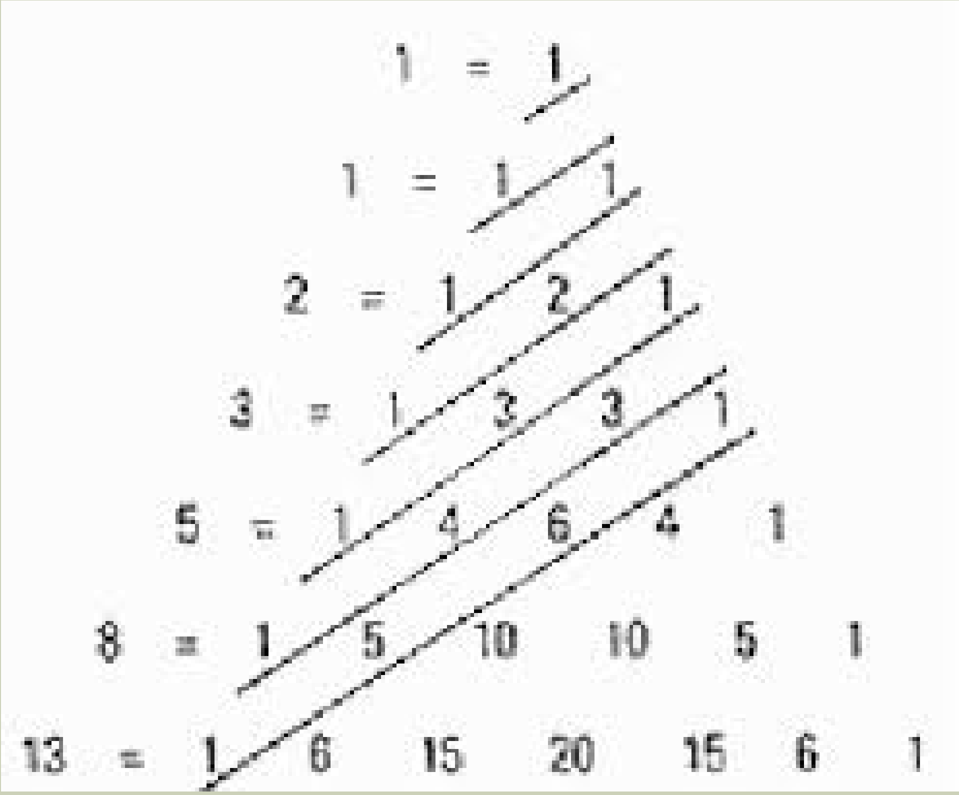
$$1 + 4 + 6 + 4 + 1 = 2^4 = 16$$

$$1 + 5 + 10 + 10 + 5 + 1 = 2^5 = 32$$

$$1 + 6 + 15 + 20 + 15 + 6 + 1 = 2^6 = 64$$

$$1 + 7 + 21 + 35 + 35 + 21 + 7 + 1 = 2^7 = 128$$

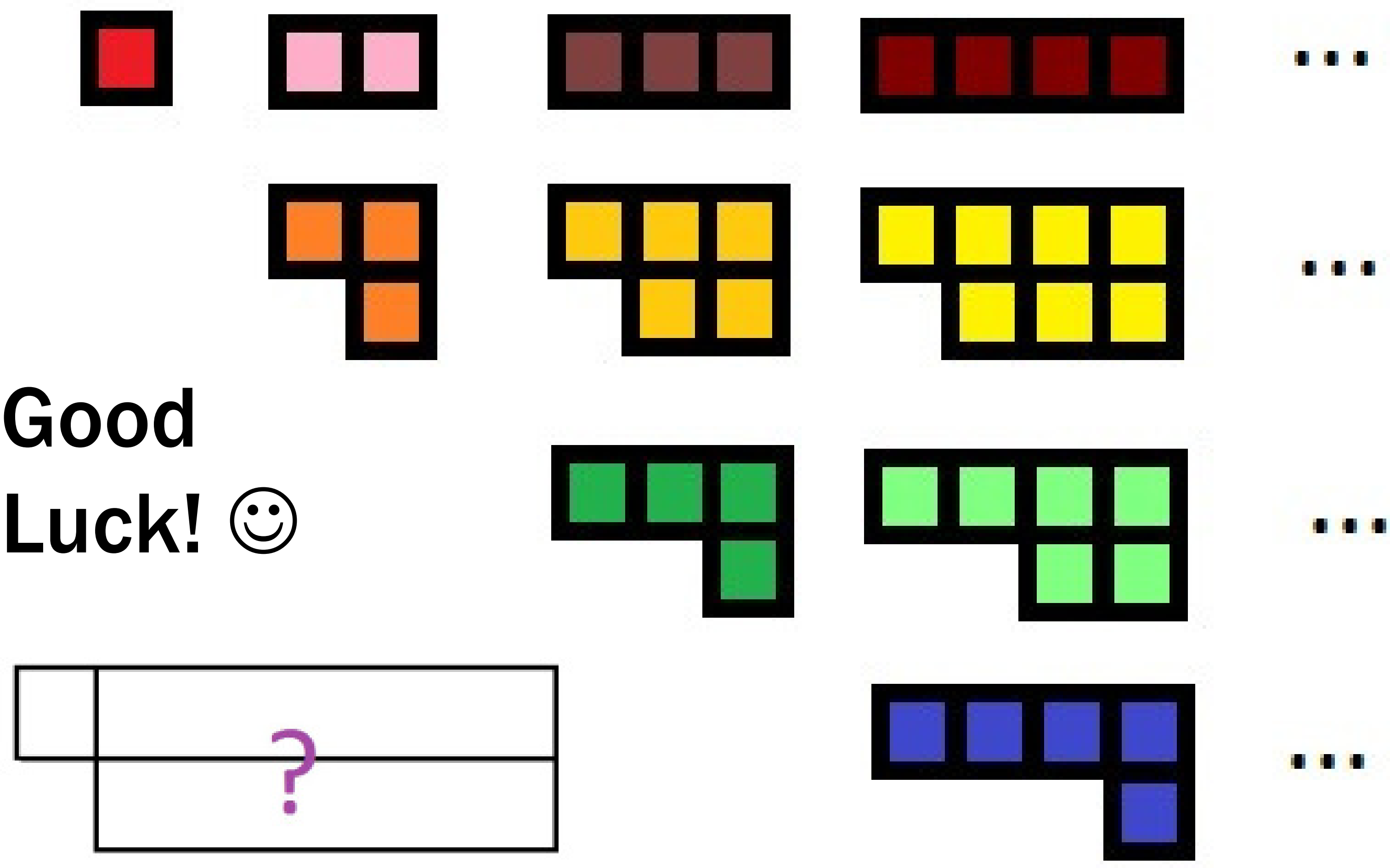
FIBONACCI IN PASCAL!



LAST BREAK FOR THE DAY:

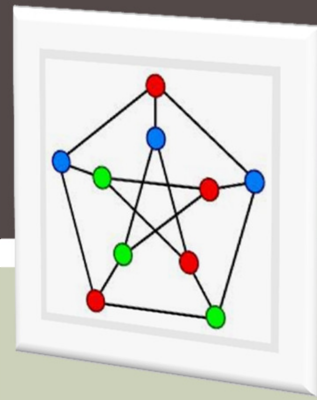
Great job everybody, that last one was pretty intense!

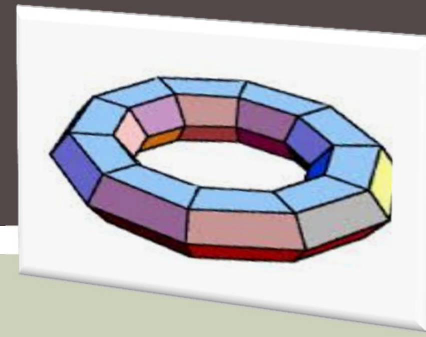
THIS ONE'S UNSOLVED



Good Luck! ☺

NEXT TOPIC





More Combinatorics!

with Graphs, Stars, and Polytopes