Problem set March 10, 2015

1. Show that $x^2 + 3y^2 = p$ is solvable if $(\frac{-3}{p}) = 1$.

2. p and q are odd primes, and $q | 2^p - 1$. Show that $q \equiv \pm 1 (\mod 8)$.

3. Let $p > 2$ be a prime, and $(a, p) = 1$. Define
$$N(a,p) := |\{(x,y) \mod p : x^2 - y^2 \equiv a(\mod p)\}|,$$
find $N(a, p)$.

4. Let $p > 2$ be a prime, and $l \geq 3$ be an integer. Show that
$$x^{2^l} \equiv 2^{2^{l-1}} (\mod p)$$
is solvable.

5. Explain (or give another proof) quadratic reciprocity in terms of Galois theory.