Applications for Bertrand's Postulate

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1) For $n \geq 2, n!$ is not a perfect square.

2)$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{n}$ is not an integer for $n = 2, 3, 4.....$

3) What's wrong with the following "easy proof" of Bertrand's Postulate?
Proof by contradiction:
Suppose for some $n \geq 3, \quad n < k < 2n \Rightarrow k$ is composit.

Let $A = \{2, 3, ..., n\}$ and $B = \{n+1, n+2, ..., 2n\}$.
Then $\mid A \mid = n - 2$ and $\mid B \mid = n - 1$.
Let function $f : B \rightarrow A$ be $f(m) = \frac{m}{p}$ where p is the smallest prime dividing m.
If f is one-one function, then we have a contradiction.

Suppose $f(v) = f(w)$.
Let $v = p_1^{a_1} p_2^{a_2} .....p_i^{a_i}$ and $w = q_1^{b_1} q_2^{b_2} .....q_j^{b_j}$
where $p_1 < p_2 < p_3 < ..... < p_i$ and $q_1 < q_2 < q_3 < ..... < q_j$.
Then $f(v) = p_1^{a_1 - 1} p_2^{a_2} .....p_i^{a_i}$ and $f(w) = q_1^{b_1 - 1} q_2^{b_2} .....q_j^{b_j}$.

Since $f(v) = f(w)$,
$i = j, p_1 = q_1, p_2 = q_2, ..., p_i = q_i$ and $a_1 = b_1, a_2 = b_2, ..., a_i = b_i$.
So $v = w$, giving us a contradiction.

4)Prove or disprove that the following graph has an Hamiltonian Circuit.

The graph has 1,000 vertices that are numbered 1,2,..., 1000. There exists an edge between two vertices iff (if and only if) the smaller number associated with one of the vertex divides the number associated with the other vertex.

* 5) $\{1, 2, ...2k\}$ can be arranged into k disjoint pairs with pair sum always a prime. E.g. {1,2,3,4,5,6} has {1,2} and {3,4} and {5,6} as three pairs because $1 + 2 = 3$ and $3 + 4 = 7$ and $5 + 6 = 11$ where 3,7, and 11 are primes.

** 6) Every integer > 6 is sum of distinct primes.

** 7) Application for computer science: Find the least integer k_n s.t. $1^2, 2^2, 3^2, ..., n^2$ are all incongruent modulo k_n for $n \geq 4$.