Exercise 1. If x is a real number, prove $x^2 \geq 0$.

Exercise 2. For any positive numbers a, b, prove that $\frac{2}{\frac{1}{a}+\frac{1}{b}} \leq \sqrt{ab} \leq \frac{a+b}{2} \leq \sqrt{\frac{a^2+b^2}{2}}$

Exercise 3. For any positive numbers $a_1, a_2, ..., a_n$, with integer $n \geq 2$, prove that $\frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+...+\frac{1}{a_n}} \leq \sqrt[n]{a_1 a_2 ... a_n} \leq \frac{a_1+a_2+...+a_n}{n} \leq \sqrt{\frac{a_1^2+a_2^2+...+a_n^2}{n}}$

Exercise 4. For any positive numbers a, b, c, prove that $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$

Exercise 5. For any real numbers x, y, z, prove that $(x+y+z)^2 \geq 3(xy+yz+zx)$

Exercise 6. For any positive numbers a, b, c, prove that $8abc \leq (a+b)(b+c)(c+a)$

Exercise 7. For any real numbers $x_1, x_2, ..., x_n$ and $y_1, y_2, ..., y_n$, with integer $n \geq 1$, show that $(\sum_{i=1}^n x_i y_i)^2 \leq (\sum_{i=1}^n x_i^2)(\sum_{i=1}^n y_i^2)$

Exercise 8. Let n be a natural number such that $n \geq 2$. Show that

$$\frac{1}{n+1}(1+\frac{1}{3}+...+\frac{1}{2n-1}) > \frac{1}{n}(\frac{1}{2}+\frac{1}{4}+...+\frac{1}{2n}).$$

Exercise 9. Suppose that the real numbers a_1,a_2,...,a_{100} satisfy $a_1 \geq a_2 \geq ... \geq a_{100} \geq 0$, $a_1 + a_2 \leq 100$, and $a_3 + a_4 + ... + a_{100} \leq 100$. Determine the maximum possible value of $a_1^2 + a_2^2 + ... + a_{100}^2$, and find all possible sequences a_1,a_2,...,a_{100} which achieve this maximum.

Exercise 10. Let the real number a,b,c,d satisfy the relations $a^2+b^2+c^2+d^2 = 12$. Prove that

$$4(a^3+b^3+c^3+d^3) - (a^4+b^4+c^4+d^4) \leq 48.$$

Exercise 11. For $t \geq s \geq 0$, $a > 0$. Show that $\frac{t}{t+a} \geq \frac{s}{s+a}$

Exercise 12. For positive numbers x, y, b, c, d, prove that $\frac{1}{1+\frac{b+d}{\sqrt{x^2+c^2}}} + \frac{1}{1+\frac{c+d}{\sqrt{y^2+b^2}}} \geq \frac{1}{1+\frac{d}{\sqrt{(x+b)^2+(y+c)^2}}}$