Russian Math Olympiads for 6th graders

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Math Olympiad

Russian Math Olympiad for 6th graders consists of two rounds. The first round is organized in schools, and the winners are determined for each of the schools separately (if the top students in a school received at least half of the maximal total number of points). The winners of the first round are invited to compete in the second (city-wide) round which has harder problems.

In both rounds a student need to present a complete solution, not just answer! Points are subtracted for making statements not supported by sufficient proofs and explanations. Today pretend you are at the Russian Math Olympiad and challenge yourself to not only solve the problems but also to write down solutions.

Math Festival

Each year, Moscow State University organizes a *Math Fest* – a special day which includes 2 hours of problem solving (contest!), as well as lectures and presentations by prominent mathematicians.

The contest is open to all students. The organizers state that it might be better *not* to know some of the standard school math!

Math Olympiad: First Round

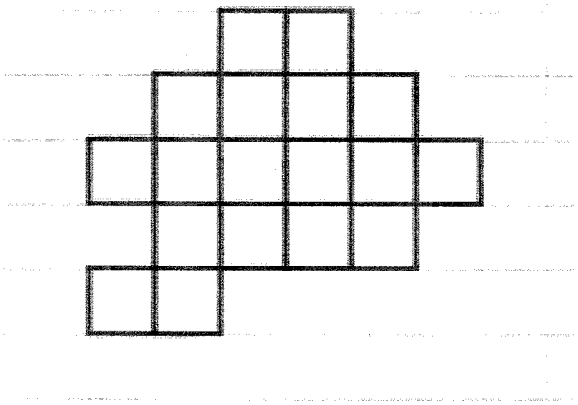
1. Peter likes to exchange stickers with his friends. For every sticker he gives someone he gets 5 stickers back. Suppose he starts the exchange with just one sticker. How many stickers will he have after 30 exchanges?

2. Compare the following numbers:

$$4 \times \underbrace{9 \times 9 \times \cdots \times 9 \times 9}_{27 \text{ times}} \quad \square \quad \underbrace{3 \times 3 \times \cdots \times 3 \times 3}_{55 \text{ times}}$$

   Explain your solution.

3. Cut the following shape into 3 parts which are equal in size and shape.



4. Three dwarves, Happy, Grumpy and Sleepy, went into a cave. One of them found a piece of gold, another – a bread mold, and the third got a head cold after touring the cave.

   - The dwarf who got head cold has a blue hat, and his beard is the longest;
   - The dwarf with the shortest beard found gold;
   - Happy has a red hat, and his beard is longer than Grumpy's;

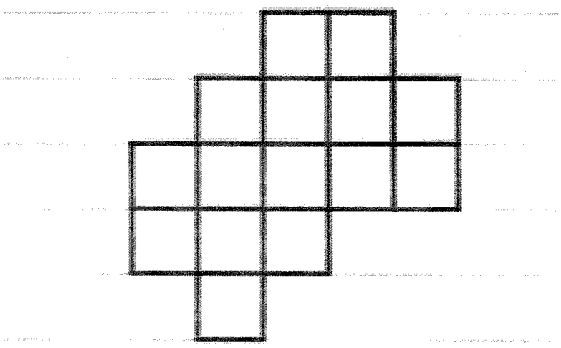
   What did each of the dwarves got after visiting the cave?

5. Write down 7 consecutive numbers so that the digit 2 is used exactly 16 times.

6. Three jumps of a two-headed dragon equal to five jumps of a three-headed dragon. It takes a two-headed dragon the same time to make 4 jumps as it takes a three-headed dragon to make 7 jumps. Which of the dragons moves faster? Explain your answer.

Math Olympiad: Second round

1. Cut the following shape into 3 equal parts.



2. There were 15 muffins on a platter. Winnie the Pooh ate 3 times as many as Eeyore. Eeyore ate 3 times as many as Owl. How many muffins are left after that?

3. Replace all the symbols $*$ by numbers so that the sum of any three numbers written next to each other is equal to 20:

$$5 \quad * \quad * \quad * \quad * \quad * \quad * \quad 8.$$

4. A boy made up his sibling's name from 4 identical cubes. The cubes are positioned so that the boy can read the name on the cubes facing him:

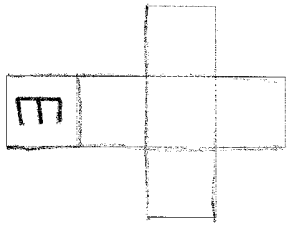


Figure out how the letters are positioned on the cubes and put the rest of the letters onto the cube net (cutout) above.

5. Two types of ladybugs live in the magic forest: some ladybugs have 6 dots and the rest of them have 4 dots each. All the ladybugs with 6 dots always tell the truth. All the ladybugs with 4 dots always lie. You met several of these ladybugs.

   - The first ladybug told you: *"All of us have the same number of dots".*
   - The second ladybug said: *"All together, we have* 30 *dots on our backs".*
   - The third ladybug said: *"No! All together, we have* 26 *dots on our backs".*

   The rest of the ladybugs each said that only one of those three ladybugs told the truth. How many ladybugs did you meet?

Mathematical Celebration

1. Ben multiplied a number by 10 and got a prime number. Peter multiplied the same number by 15 and also got a prime number. Could it be that both of them did their computations correctly? Explain your answer.

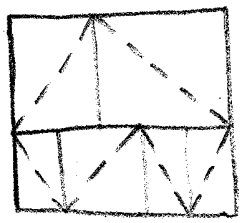
2. Solve the following riddle:

   Here is a riddle written on a cup:
   Eh *is four times as much as* **Oi**,
   Oh *is four times as little as* **Ai**,
   What do you get if you add all four of them up?

3. A dog and a cat are pulling a sausage into two different directions. If the dog takes its piece and runs away, the cat will get 300gr more than the dog. If the cat takes its piece and runs away, the dog will get 500gr more than the cat. How much of the sausage will be left if each of them takes its piece and runs away?

4. Thirteen children were sitting around the table. All the girls agreed that they will only tell the truth to each other and will lie to the boys. All the boys agreed that they will only tell the truth to each other and lie to the girls. One of the children said to his/her neighbor on the right: "*The majority of us are boys*". The neighbor told his/her neighbor on the right "*The majority of us are girls*", and so on, with the last child telling the first one: "*The majority of us are boys*". How many boys were there at the table?

5. *The Big Island* and *The Small Island* are both rectangular in shape and are divided into several rectangular counties. Each county has a road along one of the rectangle's diagonals. On each of the islands these roads form a closed path which does not go twice through any of the points. Here is a map of the *Small Island*:



Draw a possible map of the *Big Island* if you know that it has an odd number of counties. How many counties does your island have?

6. Thirty three giants are guarding a cave. The Wicked Witch agreed to pay them 240 gold coins under the following conditions:

   - The Wicked Witch divides the giants into several troops and pays each of the troops separately;
   - Within each of the troops, the coins are divided equally between the giants, and the remainder is given back to the Wicked Witch.

   (a) What is the biggest number of coins that the Wicked Witch can guarantee to herself if she has to pay each of the troops equally (independently of how many people are in each of the troops)?

   (b) What if she can give different troops different number of coins?

Other challenge problems

1. Two hikers start walking at the same time with constant (but different!) speeds. The first one walks from A to B while the second one walks from B to A. After they meet somewhere in between, it takes the first hiker 9 hours to get to city B, while it takes 16 hours for the second hiker to get to city A. How soon after the beginning of the journey did they meet?

2. The following problem is attributed to Sir Isaac Newton:

   70 cows eat the grass on a field in 24 days. 60 cows eat the grass on the same field in 30 days. How many cow would take to eat all the grass in 96 days? (Hint: the grass continues to grow at a constant rate while the cows are eating it).