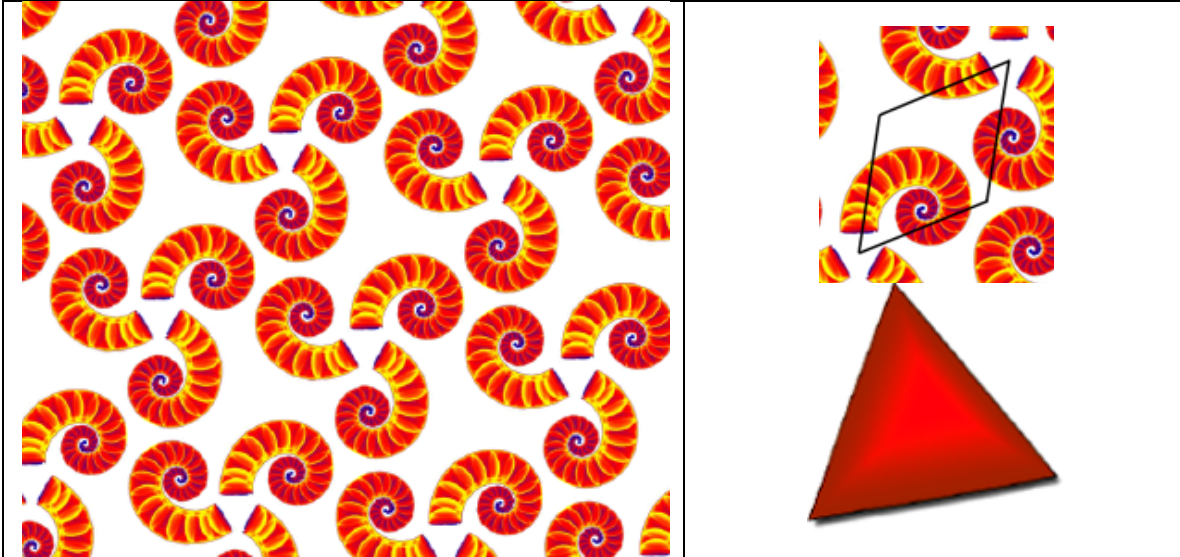


Wallpaper patterns with rotational symmetry and/ or mirror lines

1. Find the mirror lines and the points of rotational symmetry (ignore differences in coloring). Draw an outline around the smallest repeating unit that can be used to generate the whole pattern.



2. For a particular wallpaper pattern printed on a piece of paper, imagine folding the paper along mirror lines and rolling it up at rotation points so that all points on the paper with the same pattern are glued together, or “identified” as a single point. This resulting glued up object is called an orbifold. For example, the orbifold corresponding to this pattern looks like a triangular pillow with three cone points.

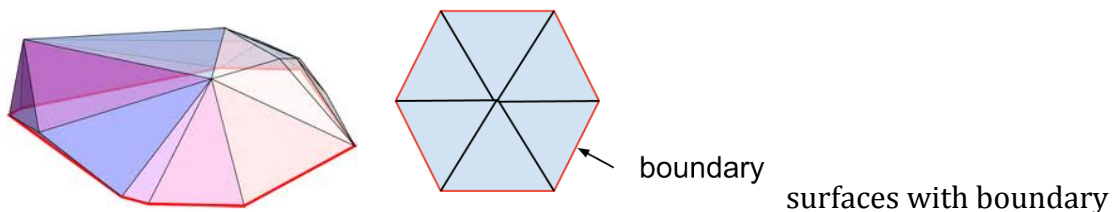


Calculate the total angle defect for the “triangular pillow” orbifold corresponding to the wallpaper pattern above.

4. Is it possible to have a wallpaper pattern with no mirror symmetry and one 8-fold, one 4-fold, and one 2-fold rotational symmetry point? Prove your answer.

5. What are all the possible symmetry patterns that have rotational symmetry but no mirror symmetry (and no “glide reflection symmetry”)? Hint: surprisingly, this is related to curvature and Descartes’ angle defect formula!

6. So far, we have only considered surfaces without boundary. State and prove a version of Descartes’ Angle Defect Formula for surfaces with boundary. Hint: be careful about how you define angle defect for vertices on the boundary.



7. Use Descartes’ angle defect formula for surfaces with boundary and the idea of an orbifold to determine the possible types of wallpaper patterns involving reflections.