**Splitting and Sharing**

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1. Is it possible to split 5 apples evenly between 6 people without cutting a single apple into more than 3 parts?
2. A rancher split his cattle between his sons as follows. The eldest son got one cow and 1/7 of the remaining cows; the next son got two cows and 1/7 of the remaining cows; the third son got three cows and 1/7 of the remaining cows, and so on. In this way, the entire herd had been split up entirely between all of the sons. How many sons did the rancher have and how large was his herd?
3. A father has 20 one dollar bills to distribute among his five sons. He declares that the oldest son will propose a scheme for dividing up the money and all five sons will vote on the plan. If a majority agree to the plan, then it will be implemented, otherwise dad will simply split the money evenly among his sons. Assume that all the sons act in a manner to maximize their monetary gain but will opt for evenly splitting the money, all else being equal. What proposal will the oldest son put forth, and why?
4. Imagine that in the scenario of the previous problem the father decides that after the oldest son’s plan is unveiled, the second son will have the opportunity to propose a different division of funds. The sons will then vote on which plan they prefer. Assume that the sons still act to maximize their monetary gain, but will vote for the older son’s plan if they stand to receive the same amount of money either way. What will transpire in this case, and why?
5. Is it possible to split a square into five squares? six squares? any other number of squares? What if we require that all these smaller squares should be of different sizes?
6. Is it possible to split an equilateral triangle into a number of equilateral triangles? For what numbers is it possible?
7. Could you ask – and answer – same questions as in problems 5 and 6, only applied to 3-dimensional objects?